

**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

Cambridge Ordinary Level

## **MARK SCHEME for the May/June 2015 series**

### **4037 ADDITIONAL MATHEMATICS**

**4037/11**

Paper 1, maximum raw mark 80

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### Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

<b>1</b>	<b>(i)</b>	180° or $\pi$ radians or 3.14 radians ( or better)	<b>B1</b>	
	<b>(ii)</b>	2	<b>B1</b>	
	<b>(iii)</b>	<b>(a)</b>	<b>B1</b>	$y = \sin 2x$ all correct
		<b>(b)</b>	<b>B1</b>	for either $\uparrow \downarrow \uparrow$ starting at their highest value and ending at their lowest value Or a curve with highest value at $y = 3$ and lowest value at $y = -1$
	<b>(iv)</b>	3	<b>B1</b>	completely correct graph
<b>2</b>	<b>(i)</b>	$\tan \theta = \frac{(8 + 5\sqrt{2})(4 - 3\sqrt{2})}{(4 + 3\sqrt{2})(4 - 3\sqrt{2})}$ $= \frac{32 - 24\sqrt{2} + 20\sqrt{2} - 30}{16 - 18}$ $= 1 + 2\sqrt{2} \text{ cao}$	<b>M1</b>	attempt to obtain $\tan \theta$ and rationalise. Must be convinced that no calculators are being used
			<b>A1</b>	

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<b>(ii)</b>	$\sec^2 \theta = 1 + \tan^2 \theta$ $= 1 + (-1 + 2\sqrt{2})^2$ $= 1 + 1 - 4\sqrt{2} + 8$ $= 10 - 4\sqrt{2}$ <p><b>Alternative solution:</b></p> $AC^2 = (4 + 3\sqrt{2})^2 + (8 + 5\sqrt{2})^2$ $= 148 + 104\sqrt{2}$ $\sec^2 \theta = \frac{148 + 104\sqrt{2}}{(4 + 3\sqrt{2})^2}$ $= \frac{148 + 104\sqrt{2}}{(4 + 3\sqrt{2})^2} \times \frac{34 - 24\sqrt{2}}{34 - 24\sqrt{2}}$ $= 10 - 4\sqrt{2}$	<b>M1</b>  <b>DM1</b>  <b>A1</b>     <b>M1</b>  <b>DM1</b>  <b>A1</b>	attempt to use $\sec^2 \theta = 1 + \tan^2 \theta$ , with <i>their</i> answer to <b>(i)</b>  attempt to simplify, must be convinced no calculators are being used.  Need to expand $(-1 + 2\sqrt{2})^2$ as 3 terms
<b>3 (i)</b>  <b>(ii)</b>	$64 + 192x^2 + 240x^4 + 160x^6$ $(64 + 192x^2 + 240x^4) \left( 1 - \frac{6}{x^2} + \frac{9}{x^4} \right)$ <p>Terms needed <math>64 - (192 \times 6) + (240 \times 9)</math></p> $= 1072$	<b>B3,2,1,0</b>  <b>B1</b>  <b>M1</b>  <b>A1</b>	-1 each error  expansion of $\left( 1 - \frac{3}{x^2} \right)^2$  attempt to obtain 2 or 3 terms using <i>their (i)</i>

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<p>4 (a)</p> <p>(b)</p>	$\mathbf{X}^2 = \begin{pmatrix} 4-4k & -8 \\ 2k & -4k \end{pmatrix}$ <p>Use of <math>\mathbf{AA}^{-1} = \mathbf{I}</math></p> $\begin{pmatrix} a & 1 \\ b & 5 \end{pmatrix} \begin{pmatrix} \frac{5}{6} & -\frac{1}{6} \\ -\frac{2}{3} & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ <p>Any 2 equations will give <math>a = 2, b = 4</math></p> <p><b>Alternative method 1:</b></p> $\frac{1}{5a-b} \begin{pmatrix} 5 & -1 \\ b & a \end{pmatrix} = \begin{pmatrix} \frac{5}{6} & -\frac{1}{6} \\ -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$ <p>Compare any 2 terms to give <math>a = 2, b = 4</math></p> <p><b>Alternative method 2:</b></p> <p>Inverse of <math>\frac{1}{6} \begin{pmatrix} 5 &amp; -1 \\ -4 &amp; 2 \end{pmatrix} = \begin{pmatrix} 2 &amp; 1 \\ 4 &amp; 5 \end{pmatrix}</math></p>	<p><b>B2,1,0</b></p> <p><b>M1</b></p> <p><b>A1,A1</b></p> <p><b>M1</b></p> <p><b>A1,A1</b></p> <p><b>M1</b></p> <p><b>A1,A1</b></p>	<p>-1 each incorrect element</p> <p>use of <math>\mathbf{AA}^{-1} = \mathbf{I}</math> and an attempt to obtain at least one equation.</p> <p>correct attempt to obtain <math>\mathbf{A}^{-1}</math> and comparison of at least one term.</p> <p>reasoning and attempt at inverse</p>
<p>5</p>	<p><math>3x - 1 = x(3x - 1) + x^2 - 4</math> or</p> $y = \left(\frac{y+1}{3}\right)y + \left(\frac{y+1}{3}\right)^2 - 4$ <p><math>4x^2 - 4x - 3 = 0</math> or <math>4y^2 - 4y - 35 = 0</math></p> <p><math>(2x - 3)(2x + 1) = 0</math> or <math>(2y - 7)(2y + 5) = 0</math></p> <p>leading to <math>x = \frac{3}{2}, x = -\frac{1}{2}</math> and</p> $y = \frac{7}{2}, y = -\frac{5}{2}$ <p>Midpoint <math>\left(\frac{1}{2}, \frac{1}{2}\right)</math></p> <p>Perpendicular gradient = <math>-\frac{1}{3}</math></p> <p>Perp bisector: <math>y - \frac{1}{2} = -\frac{1}{3}\left(x - \frac{1}{2}\right)</math></p> <p><math>(3y + x - 2 = 0)</math></p>	<p><b>M1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>equate and attempt to obtain an equation in 1 variable</p> <p>forming a 3 term quadratic equation and attempt to solve</p> <p><math>x</math> values</p> <p><math>y</math> values</p> <p>for midpoint, allow anywhere</p> <p>correct attempt to obtain the gradient of the perpendicular, using <math>AB</math></p> <p>straight line equation through the midpoint; must be convinced it is a perpendicular gradient.</p> <p>allow unsimplified</p>

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<p><b>6 (i)</b></p> $f\left(\frac{1}{2}\right) = \frac{a}{8} - \frac{15}{4} + \frac{b}{2} - 2 = 0$ <p>leading to <math>a + 4b = 46</math>  <math>f(1) = a - 15 + b - 2 = 5</math>  leading to <math>a + b = 22</math></p> <p>giving <math>b = 8</math> (AG), <math>a = 14</math></p> <p><b>(ii)</b></p> $(2x - 1)(7x^2 - 4x + 2)$ <p><b>(iii)</b></p> $7x^2 - 4x + 2 = 0 \text{ has no real solutions as}$ $b^2 < 4ac$ $16 < 56$		<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1,A1</b></p> <p><b>M1,A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>correct use of either <math>f\left(\frac{1}{2}\right)</math> or <math>f(1)</math>  paired correctly</p> <p>both equations correct (allow unsimplified)</p> <p>M1 for solution of equations  A1 for both <math>a</math> and <math>b</math>. <b>AG</b> for <math>b</math>.</p> <p>M1 for valid attempt to obtain <math>g(x)</math>, by either observation or by algebraic long division.</p> <p>use of <math>b^2 - 4ac</math></p> <p>correct conclusion; must be from a correct <math>g(x)</math> or <math>2g(x)</math> www</p>
<p><b>7 (i)</b></p> $\frac{dy}{dx} = \frac{(x-1)\left(\frac{8x}{4x^2+2}\right) - \ln(4x^2+3)}{(x-1)^2}$ <p>When <math>x = 0</math>, <math>y = -\ln 3</math> oe</p> $\frac{dy}{dx} = -\ln 3 \text{ so gradient of normal is } \frac{1}{\ln 3}$ <p>(allow numerical equivalent)</p> <p>normal equation <math>y + \ln 3 = \frac{1}{\ln 3}x</math>  or <math>y = 0.910x - 1.10</math>, or <math>y = \frac{10}{11}x - \frac{11}{10}</math> cao  (Allow <math>y = 0.91x - 1.1</math>)</p> <p><b>(ii)</b></p> <p>when <math>x = 0</math>, <math>y = -\ln 3</math>  when <math>y = 0</math>, <math>x = (\ln 3)^2</math>  Area = <math>\pm 0.66</math> or <math>\pm 0.67</math> or awrt these  or <math>\frac{1}{2}(\ln 3)^3</math></p>		<p><b>M1</b></p> <p><b>B1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>differentiation of a quotient (or product)</p> <p>correct differentiation of <math>\ln(4x^2 + 3)</math></p> <p>all else correct</p> <p>for <math>y</math> value</p> <p>valid attempt to obtain gradient of the normal</p> <p>attempt at normal equation must be using a perpendicular</p> <p>valid attempt at area</p>

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8	(i)	Range for f: $y \geq 3$ Range for g: $y \geq 9$	<b>B1</b> <b>B1</b>	
	(ii)	$x = -2 + \sqrt{y-5}$ $g^{-1}(x) = -2 + \sqrt{x-5}$ Domain of $g^{-1}$ : $x \geq 9$  <b>Alternative method:</b> $y^2 + 4y + 9 - x = 0$ $y = \frac{-4 + \sqrt{16 - 4(9-x)}}{2}$	<b>M1</b> <b>A1</b> <b>B1</b>	attempt to obtain the inverse function  Must be correct form for domain
	(iii)	Need $g(3e^{2x})$ $(3e^{2x} + 2)^2 + 5 = 41$ or $9e^{4x} + 12e^{2x} - 32 = 0$ $(3e^{2x} - 4)(3e^{2x} + 8) = 0$ leading to $3e^{2x} + 2 = \pm 6$ so $x = \frac{1}{2} \ln \frac{4}{3}$  or $e^{2x} = \frac{4}{3}$ so $x = \frac{1}{2} \ln \frac{4}{3}$  <b>Alternative method:</b> Using $f(x) = g^{-1}(41)$ , $g^{-1}(41) = 4$ leading to $3e^{2x} = 4$ , so $x = \frac{1}{2} \ln \frac{4}{3}$	<b>M1</b> <b>DM1</b>  <b>M1</b>  <b>A1</b>  <b>M1</b> <b>DM1</b>  <b>M1</b> <b>A1</b>	correct order correct attempt to solve the equation  dealing with the exponential correctly in order to reach a solution for $x$  Allow equivalent logarithmic forms  correct use of $g^{-1}$ dealing with $g^{-1}(41)$ to obtain an equation in terms of $e^{2x}$ dealing with the exponential correctly in order to reach a solution for $x$ Allow equivalent logarithmic forms
	(iv)	$g'(x) = 6e^{2x}$ $g'(\ln 4) = 96$	<b>B1</b> <b>B1</b>	B1 for each

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<p><b>9 (i)</b></p> $\frac{dy}{dx} = 3x^2 - 10x + 3$ <p>When <math>x = 0</math>, for curve <math>\frac{dy}{dx} = 3</math>, gradient of line also 3 so line is a tangent.</p> <p><b>Alternate method:</b> <math>3x + 10 = x^3 - 5x^2 + 3x + 10</math></p> <p>leading to <math>x^2 = 0</math>, so tangent at <math>x = 0</math></p> <p><b>(ii)</b></p> <p>When <math>\frac{dy}{dx} = 0</math>, <math>(3x - 1)(x - 3) = 0</math> <math>x = \frac{1}{3}, x = 3</math></p> <p><b>(iii)</b></p> $\text{Area} = \frac{1}{2}(10 + 19)3 - \int_0^3 x^3 - 5x^2 + 3x + 10 dx$ $= \frac{87}{2} - \left[ \frac{x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} + 10x \right]_0^3$ $= \frac{87}{2} - \left( \frac{81}{4} - 45 + \frac{27}{2} + 30 \right)$ $= 24.7 \text{ or } 24.8$ <p><b>Alternative method:</b> <math>\text{Area} = \int_0^3 (3x + 10) - (x^3 - 5x^2 + 3x + 10) dx</math></p> $= \int_0^3 -x^3 + 5x^2 dx$ $= \left[ -\frac{x^4}{4} + \frac{5x^3}{3} \right]_0^3 = \frac{99}{4}$	<p><b>M1</b> for differentiation</p> <p><b>A1</b> comparing both gradients</p> <p><b>M1</b> attempt to deal with simultaneous equations</p> <p><b>A1</b> obtaining <math>x = 0</math></p> <p><b>M1</b> equating gradient to zero and valid attempt to solve</p> <p><b>A1,A1</b> A1 for each</p> <p><b>B1</b> area of the trapezium</p> <p><b>M1</b> attempt to obtain the area enclosed by the curve and the coordinate axes, by integration</p> <p><b>A1</b> integration all correct</p> <p><b>DM1</b> correct application of limits (must be using <i>their</i> 3 from <b>(ii)</b> and 0)</p> <p><b>A1</b></p> <p><b>B1</b> correct use of 'Y-y'</p> <p><b>M1</b> attempt to integrate</p> <p><b>A1</b> integration all correct</p> <p><b>DM1</b> correct application of limits</p> <p><b>A1</b></p>	
<p><b>10 (a)</b></p> $\sin^2 x = \frac{1}{4}$ $\sin x = (\pm) \frac{1}{2}$ <p><math>x = 30^\circ, 150^\circ, 210^\circ, 330^\circ</math></p>	<p><b>M1</b></p> <p><b>A1,A1</b></p>	<p>using <math>\operatorname{cosec} x = \frac{1}{\sin x}</math> and obtaining <math>\sin x = \dots</math></p> <p>A1 for one correct pair, A1 for another correct pair with no extra solutions</p>

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<p><b>(b)</b></p> $(\sec^2 3y - 1) - 2\sec 3y - 2 = 0$ $\sec^2 3y - 2\sec 3y - 3 = 0$ $(\sec 3y + 1)(\sec 3y - 3) = 0$ <p>leading to <math>\cos 3y = -1</math>, <math>\cos 3y = \frac{1}{3}</math></p> $3y = 180^\circ, 540^\circ \quad 3y = 70.5^\circ, 289.5^\circ, 430.5^\circ$ $y = 60^\circ, 180^\circ, 23.5^\circ, 96.5^\circ, 143.5^\circ$ <p><b>Alternative 1:</b></p> $\sec^2 3y - 2\sec 3y - 3 = 0$ <p>leading to <math>3\cos^2 3y + 2\cos 3y - 1</math></p> $(3\cos y - 1)(\cos y + 1) = 0$ <p><b>Alternative 2:</b></p> $\frac{\sin^2 y}{\cos^2 y} - \frac{2}{\cos y} - 2 = 0$ $(1 - \cos^2 x) - 2\cos x - 2\cos^2 x = 0$		<p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1,A1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p>	<p>use of the correct identity</p> <p>attempt to obtain a 3 term quadratic equation in <math>\sec 3y</math> and attempt to solve dealing with <math>\sec</math> and <math>3y</math> correctly</p> <p>A1 for a correct pair, A1 for a second correct pair, A1 for correct 5<sup>th</sup> solution and no other within the range</p> <p>use of the correct identity</p> <p>attempt to obtain a quadratic equation in <math>\cos 3y</math> and attempt to solve dealing with <math>3y</math> correctly</p> <p>A marks as above</p> <p>use of the correct identity,  <math>\tan y = \frac{\sin y}{\cos y}</math> and <math>\sec y = \frac{1}{\cos y}</math>, then  as before</p>
<p><b>(c)</b></p> $z - \frac{\pi}{3} = \frac{\pi}{3}, \frac{4\pi}{3}$ $z = \frac{2\pi}{3}, \frac{5\pi}{3} \text{ or } 2.09 \text{ or } 2.1, 5.24$		<p><b>M1</b></p> <p><b>A1,A1</b></p>	<p>correct order of operations</p> <p>A1 for a correct solution  A1 for a second correct solution and no other within the range</p>