Example Candidate Responses

Cambridge International AS & A Level

Cambridge International AS and A Level Mathematics

9709

Paper 4

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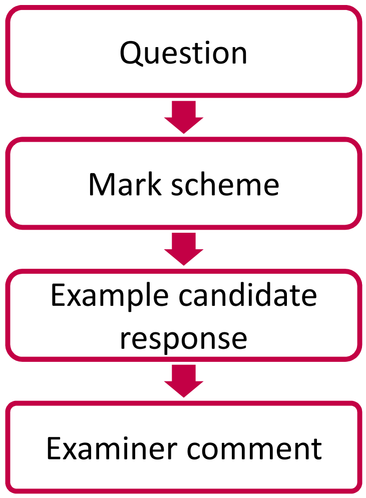
Contents

Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics (9709), and to show how different levels of candidates' performance relate to the subject's curriculum and assessment objectives.

In this booklet candidate responses have been chosen to exemplify a range of answers. Each response is accompanied by a brief commentary explaining the strengths and weaknesses of the answers.

For ease of reference the following format for each component has been adopted:



Each question is followed by an extract of the mark scheme used by examiners. This, in turn, is followed by examples of marked candidate responses, each with an examiner comment on performance. Comments are given to indicate where and why marks were awarded, and how additional marks could have been obtained. In this way, it is possible to understand what candidates have done to gain their marks and what they still have to do to improve them.

Past papers, Examiner Reports and other teacher support materials are available on Teacher Support at https://teachers.cie.org.uk

Assessment at a glance

The 7 units in the scheme cover the following subject areas:

- Pure Mathematics (units P1, P2 and P3);
- Mechanics (units M1 and M2);
- Probability and Statistics (units S1 and S2).

Centres and candidates may:

- take all four Advanced (A) Level components in the same examination session for the full A Level.
- follow a staged assessment route to the A Level by taking two Advanced Subsidiary (AS) papers (P1 & M1 **or** P1 & S1) in an earlier examination session;
- take the Advanced Subsidiary (AS) qualification only.

AS Level candidates take:

Paper 1: Pure Mathematics 1 (P1)
1¾ hours About 10 shorter and longer questions 75 marks weighted at 60% of total

plus **one** of the following papers:

Paper 2: Pure Mathematics 2 (P2)	**Paper 4: Mechanics 1 (M1)**	**Paper 6: Probability and Statistics (S1)**
1¼ hours About 7 shorter and longer questions 50 marks weighted at 40% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 40% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 40% of total

A Level candidates take:

Paper 1: Pure Mathematics 1 (P1)	Paper 3 Pure Mathematics 3 (P3)
1¾ hours About 10 shorter and longer questions 75 marks weighted at 30% of total	**1¾ hours** About 10 shorter and longer questions 75 marks weighted at 30% of total

plus **one** of the following combinations of two papers:

Paper 4: Mechanics 1 (M1)	Paper 6: Probability and Statistics 1 (S1)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 20% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 20% of total

or

Paper 4: Mechanics 1 (M1)	Paper 5: Mechanics 2 (M2)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 20% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 20% of total

or

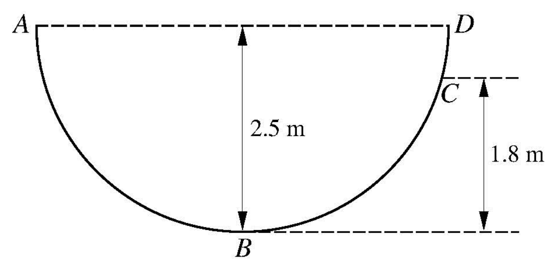
Paper 6: Probability and Statistics 1 (S1)	Paper 7: Probability and Statistics 2 (S2)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 20% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 20% of total

Teachers are reminded that the latest syllabus is available on our public website at **www.cie.org.uk** and Teacher Support at **https://teachers.cie.org.uk**

Paper 4

Question 1

1



ABCD is a semi-circular cross-section, in a vertical plane, of the inner surface of half a hollow cylinder of radius 2.5 m which is fixed with its axis horizontal. *AD* is horizontal, *B* is the lowest point of the cross-section and *C* is at a height of 1.8 m above the level of *B* (see diagram). A particle *P* of mass 0.8 kg is released from rest at *A* and comes to instantaneous rest at *C*.

(i) Find the work done on *P* by the resistance to motion while *P* travels from *A* to *C*. [2]

The work done on *P* by the resistance to motion while *P* travels from *A* to *B* is 0.6 times the work done while *P* travels from *A* to *C*.

(ii) Find the speed of *P* when it passes through *B*. [3]

Mark scheme

1	**(i)**	PE loss = $0.8g \times (2.5 - 1.8)$ (= 5.6J)	B1		
		Work done is 5.6 J	B1	2	
	(ii)		M1		For using KE gain = PE loss – WD against resistance
		$\frac{1}{2}\, 0.8v^2 = 0.8g \times 2.5 - 0.6 \times 5.6$	A1ft		
		Speed at *B* is 6.45 ms^{-1}	A1	3	

Example candidate response – 1

1\. (i) WD by R = loss in E

= mgh at AB − mgh at BC

= $0.8(10)(2.5) - 0.8(10)(1.8)$

= $20 - 14.4$

[2] = 5.6 J ✓

(ii) KE = ~~KE~~ WD by R

$\frac{1}{2}mv^2 = 0.6(5.6)$

[M0] $\frac{1}{2} \times 0.8 \times v^2 = 3.36$

$v^2 = \frac{3.36 \times 2}{0.8}$

[A0]

[A0] $v^2 = 8.4$

$v = 2.90\ ms^{-1}$

[2]

Item marks awarded: (i) = 2/2; (ii) = 0/3

Total mark awarded = 2 out of 5

Example candidate response – 2

1\. i) ME at A:

$PE = mgh$

$= (0.8)(10)(2.5)$

$= 20\ J$ ✓

ME at C:

$PE = mgh$

$= (0.8)(10)(1.8)$

$= 14.4\ J$ ✓

work done = ME at A + ME at C

0 $= 20 + 14.4$

$= 34.4\ J$

ii) work done AB = 0.6 work done AC

$= 0.6\ (34.4)$

$= 20.6\ J$

work done AB = PE + KE

$20.6 = mgh + \frac{1}{2}mv^2$

M1 $20.6 = 20 + \frac{1}{2}(0.8)v^2$

A0 $0.6 = 0.4v^2$

A0 $v^2 = 1.5$

(1) $v = 1.22\ ms^{-1}$

Item marks awarded: (i) = 0/2; (ii) = 1/3

Total mark awarded = 1 out of 5

Examiner comment – 1 and 2

(i) This question requires candidates to calculate the potential energy loss between A and C.

Candidate 1 has calculated the difference between the potential energies at A and at C and correctly obtained 5.6 J for the work done, thus gaining both marks.

Candidate 2 has found the potential energy at A and at C but has calculated the total rather than the difference. This scores no marks.

(ii) Both candidates have attempted to form a work / energy equation.

Candidate 1 includes work done and kinetic energy gained, but omits to consider the potential energy lost. The equation formed contains two instead of three terms and the answer 2.90 ms^{-1} is thus incorrect.

Candidate 2 forms an equation combining the three terms KE gain, PE loss and work done against resistance using 0.6 × the value obtained in part **(i)** for the work done from A to C. This gains a method mark.

The equation formed shows the work done as the sum instead of the difference between PE loss and KE gain. The equation and the solution 1.22 ms^{-1} are therefore both incorrect.

Question 2

2 A particle moves in a straight line. Its velocity t seconds after leaving a fixed point O on the line is $v\,\mathrm{m\,s^{-1}}$, where $v = 0.2t + 0.006t^2$. For the instant when the acceleration of the particle is 2.5 times its initial acceleration,

(i) show that $t = 25$, [3]

(ii) find the displacement of the particle from O. [3]

Mark scheme

2	**(i)**	$[a = 0.2 + 0.012t]$	M1		For differentiating to find $a(t)$.
		$[0.2 + 0.012t = 2.5 \times 0.2]]$	M1		For attempting to solve $a(t) = 2.5a(0)$
		$t = 25$	A1	3	AG
	(ii)	$[s = 0.1t^2 + 0.002t^3 \ \ (+ C)]$	M1		For integrating to find $s(t)$
		$[s = 0.1 \times 625 + 0.002 \times 15625]$	DM1		For using limits 0 to 25 or evaluating $s(t)$ with $C = 0$ (which may be implied by its absence)
		Displacement is 93.75 (accept 93.7 or 93.8)	A1	3	

Example candidate response – 1

2\. (i) $V = 0.2t + 0.006t^2$

$\frac{dv}{dt} = 0.2 + 0.012t$ ✓

when $t = 0$,

$a = \frac{dv}{dt} = 0.2 + 0.012(0)$

$= 0.2$ ✓

when $a = 2.5(0.2)$

$= 0.5$, ✓

$\Rightarrow a = 0.2 + 0.012t$

$0.5 = 0.2 + 0.012t$

$0.3 = 0.012t$

$t = 25\ s$ (shown) ✓ 3

(ii) $\int v = \int_0^{25} 0.2t + 0.006t^2\, dt$

$s = \left[\frac{0.2t^2}{2} + \frac{0.006t^3}{3}\right]_0^{25}$ ✓ M1

$= [0.1t^2 + 0.002t^3]_0^{25}$ M1

$= (62.5 + 31.25) - 0$

$= 31.25\ m$ A0

5

Item marks awarded: (i) = 3/3; (ii) = 2/3

Total mark awarded = 5 out of 6

Example candidate response – 2

2. (i) $\frac{dv}{dt} = 0.2 + 0.012t = a$ ✓

M1 when $a = 0$, $t = \frac{0.2}{0.012}$

$= 16.67$

when $v = 0$, $0.2t + 0.006t^2 = 0$

$0.006t^2 = -0.2t$

$t = \frac{-0.2}{0.006}$

$= 33.33$

$u = 0$, $v = at$

$0.2t + 0.006t^2 = (0.2 + 0.012t)t$

M0 $0.2t + 0.006t^2 = 0.2t^2 + 0.012t^2$

A0

2. (ii) $\int v =$ ~~0.2t~~ $0.1t^2 + 0.002t^3$ ✓ $= S$, M1

$s = 0.1(25^2) + 0.002(25^3)$ M1

$= 1593.8\,m$ A0

(3)

Item marks awarded: (i) = 1/3; (ii) = 2/3

Total mark awarded = 3 out of 6

Examiner comment – 1 and 2

(i) Both candidates have differentiated to find an expression for the acceleration and have gained the first method mark.

Candidate 1 substitutes $t = 0$ to obtain the initial acceleration, forms a suitable equation $(0.5 = 0.2 + 0.12t)$ and solves it to obtain $t = 25$, as given in the question.

Candidate 2 considers $a = 0$, $v = 0$ and $u = 0$ in three attempts to find the required value of t. The candidate did not realise that 'initial' acceleration means that $t = 0$ and thus makes no further progress.

(ii) Both candidates have used integration to find the displacement in terms of t and have gained the first method mark.

Candidate 1 applies both limits of $t = 25$ and $t = 0$ and calculates the difference. The calculation work contains an error $(62.5 + 31.25 = 31.25)$ so the accuracy mark was not awarded.

Candidate 2 shows the substitution of $t = 25$ into the expression for displacement but does not show substitution of $t = 0$ which has a zero value. This is acceptable but the calculation contains an error ($t3$ used in place of $t2$) leading to 1593.8 m which results in the accuracy mark not being awarded.

Question 3

3 A particle P is projected vertically upwards, from a point O, with a velocity of $8\,\text{m}\,\text{s}^{-1}$. The point A is the highest point reached by P. Find

(i) the speed of P when it is at the mid-point of OA, [4]

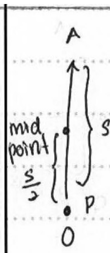
(ii) the time taken for P to reach the mid-point of OA while moving upwards. [2]

Mark scheme

3	**(i)**	$[0 = 8^2 - 2gs]$	M1		For using $0 = u^2 - 2gs$
		Maximum height is 3.2 m	A1		
		$[v^2 = 8^2 - 2g \times 1.6]$	M1		For using $v^2 = u^2 - 2gs$
		Speed is 5.66 ms^{-1}	A1	4	
	(ii)	$[5.65685... = 8 - 10t]$	M1		For using $v = u - gt$
		Time is 0.234 s	A1	2	

Example candidate response – 1

3. i)



distance OA :

$v = 0$, $u = 8$, $a = -10$

$v^2 = u^2 + 2as$

$0 = 8^2 + 2(-10)s$

$20s = 64$

$s = 3.2$ ✓

distance of mid-point OA :

$\frac{3.2}{2} = 1.6\ m$ ✓

∴ speed at mid-point OA :

$v^2 = u^2 + 2as$

$v^2 = 8^2 + 2(-10)(1.6)$

$v^2 = 64 + 32$

$v^2 = 32$

$v = 5.66\ ms^{-1}$ # ✓

4

ii) $s = 1.6\ m$

$v = 5.66\ ms^{-1}$

$u = 8\ ms^{-1}$

$a = -10$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$1.6 = 8t + \frac{1}{2}(-10)t^2$

$1.6 = 8t - 5t^2$

$5t^2 - 8t + 1.6 = 0$ ✓

$t = 1.37s$; $t = 0.23s$

∴ time taken for P to reach mid-point of OA = 1.37s #

M1

A0

(5)

Item marks awarded: (i) = 4/4; (ii) = 1/2

Total mark awarded = 5 out of 6

Example candidate response – 2

3. (i) KE gain = PE loss

$\frac{1}{2}m(v^2-u^2) = mgh$

$\frac{1}{2} \times m \times (0^2-8^2) = m \times 10 \times h$

$\frac{1}{2} \times m \times 64 = 10mh$

$32m = 10mh$

$h = 3.2m$ ✓

$v^2 = u^2 + 2as$

$= (8)^2 + 2(10)(3.2)$

$= 128$

$V = 11.3 \text{ ms}^{-1}$

M1 A0 M0 A0

(ii) $v = u + at$

$11.3 = 8 + (10)t$

$3.3 = 10t$

$t = 0.33s$

M1 A0

(2)

Item marks awarded: (i) = 1/4; (ii) = 1/2

Total mark awarded = 2 out of 6

Examiner comment – 1 and 2

(i) This part of the question could be solved using constant acceleration formulae or with energy equations.

Candidate 1 has applied $v^2 = u^2 + 2as$, firstly to obtain the distance OA and secondly to obtain the speed at the midpoint of OA. The speed 5.66 is rounded to the required three significant figures and the solution gains full marks.

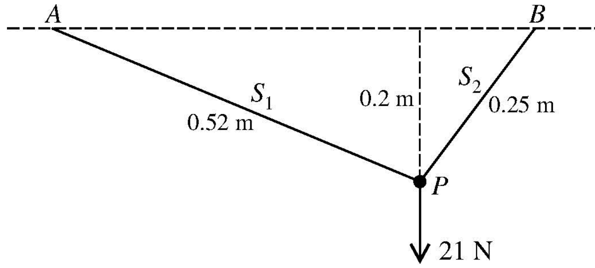
Candidate 2 has attempted to equate kinetic and potential energies to find the distance OA, but the solution contains some inconsistent sign work. The equation set up as $\frac{1}{2}m(0^2 - 8^2) = m \times 10 \times h$ leads to $h = -3.2$ rather than $h = 3.2$ as stated. The candidate then omits to halve the distance found before calculating the speed at the midpoint of OA.

(ii) Candidate 1 has formed a correct quadratic equation using $s = ut + \frac{1}{2}at^2$ and solved this to obtain two times (1.37 s and 0.23 s). Since the question asks for the time 'moving up' to the midpoint, the chosen answer of 1.37 is incorrect. If the candidate chose 0.23, the accuracy mark would still not be gained because this is a two significant figure answer instead of three significant figures.

Candidate 2's solution using $v = u + at$ shows an easier and more commonly used method. This solution using 'g' instead of '$-g$' highlights the need for candidates to check the direction of motion and consider if each value is positive or negative.

Question 4

4

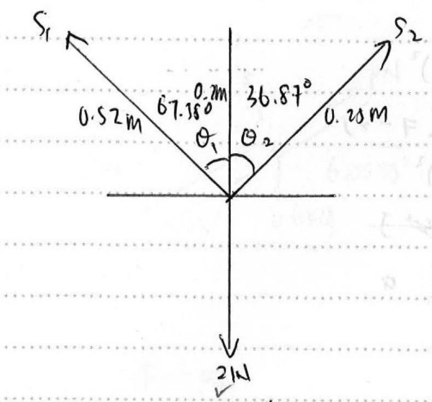


A particle P of weight 21 N is attached to one end of each of two light inextensible strings, S_1 and S_2, of lengths 0.52 m and 0.25 m respectively. The other end of S_1 is attached to a fixed point A, and the other end of S_2 is attached to a fixed point B at the same horizontal level as A. The particle P hangs in equilibrium at a point 0.2 m below the level of AB with both strings taut (see diagram). Find the tension in S_1 and the tension in S_2. [6]

4	[$T_1\sin APN = T_2\sin BPN$]	M1		For resolving forces horizontally
	$(12 \div 13)T_1 = (15 \div 25)T_2$ or $T_1\sin 67.4^o = T_2\sin 36.9^o$	A1		AEF
	[$T_1\cos APN + T_2\cos BPN = 21$]	M1		For resolving forces vertically
	$(5 \div 13)T_1 + (20 \div 25)T_2 = 21$ or $T_1\cos 67.4^o + T_2\cos 36.9^o = 21$	A1		AEF
		M1		For solving for T_1 and T_2
	Tension in S_1 is 13 N, tension in S_2 is 20 N	A1	6	
	Alternative solution using Lami's Theorem			
4	[$T_1/\sin(180 - BPN) = 21/\sin(APN + BPN)$]	M1		For using Lami's Theorem to form an equation in T_1
	$T_1/\sin(180 - \cos^{-1}(20/25))=$ $21/\sin(\cos^{-1}(20/25) + \cos^{-1}(20/52))$ or $T_1/\sin(180 - 36.9) = 21/\sin(36.9 + 67.4)$	A1		AEF
	[$T_2/\sin(180 - APN) = 21/\sin(APN + BPN)$]	M1		For using Lami's Theorem to form an equation in T_2
	$T_2/\sin(180 - \cos^{-1}(20/52)) =$ $21/\sin(\cos^{-1}(20/25) + \cos^{-1}(20/52))$ or $T_2/\sin(180-67.4)=21/\sin(36.9 + 67.4)$	A1		AEF
		M1		For solving for T_1 and T_2
	Tension in S_1 is 13 N, tension in S_2 is 20 N	A1	6	
	Alternative solution using Sine Rule			
4	[$T_1/\sin BPN = 21/\sin(180 -(APN + BPN))$]	M1		For using the Sine Rule on a triangle of forces to form an equation in T_1
	$T_1/(15/25) = 21/\sin(\cos^{-1}(20/25) + \cos^{-1}(20/52))$ or $T_1/\sin 36.9^o = 21/\sin(180 -(36.9 + 67.4))$	A1		AEF
	[$T_2/\sin APN = 21/\sin(180 -(APN + BPN))$]	M1		For using the Sine Rule to form an equation in T_2
	$T_2/(12/13) = 21/\sin(\cos^{-1}(20/25) + \cos^{-1}(20/52))$ or $T_2/\sin 67.4^0 = 21/\sin(180 -(36.9 + 67.4))$	A1		AEF
		M1		For solving for T_1 and T_2
	Tension in S_1 is 13 N, tension in S_2 is 20 N	A1	6	

Example candidate response – 1

(4)



~~θ_1~~ → $\cos\theta_1 = \frac{\text{opposite}}{\text{hypotenuse}}$ $\frac{0.2}{0.52}$

$\theta_1 = 67.38^\circ$ ✓

→ $\cos\theta_2 = \frac{0.2}{0.25}$ ✓

$\theta_2 = 36.87^\circ$

→ horizontal line ⇒ $S_1 \sin 67.38^\circ = S_2 \sin 36.87^\circ$ M1

$S_1 = \frac{S_2 \sin 36.87^\circ}{\sin 67.38^\circ}$ ⟶ ① A1

→ ② $N = S_1 \cos 67.38^\circ + S_2 \cos 36.87^\circ$ M1

~~$2 = S \cos 67.38^\circ$~~ A0

$2 = S_2\left(\frac{\sin 36.87^\circ}{\sin 67.38^\circ}\right) + S_2 \cos 36.87^\circ$

~~$2 = 0.65S_2 + 0.80S_2$~~ M1

$2 = 1.45 S_2$ A0

$S_2 = \underline{\underline{1.38\ N}}$

→ when $S_2 = 1.38$ N, $S_1 = S_2\left(\frac{\sin 36.87^\circ}{\sin 67.38^\circ}\right)$

$= 1.38\left(\frac{\sin 36.87^\circ}{\sin 67.38^\circ}\right)$

$S_1 = \underline{\underline{0.90\ N}}$ # (4)

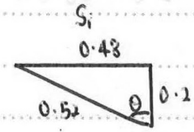
∴ $S_1 = 0.90$ N and $S_2 = 1.38$ N #

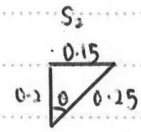
Total mark awarded = 4 out of 6

Example candidate response – 2

4. (→) $S_1 \sin\theta = S_2 \sin\theta$

(↑) $S_1 \cos\theta + S_2 \cos\theta = 21$





$\cos\theta = \frac{0.52}{0.2}$ $\cos\theta = \frac{0.25}{0.2}$

$\sin\theta = \frac{0.48}{0.2}$ $\sin\theta = \frac{0.15}{0.2}$

M1 A0

(→) $S_1\left(\frac{0.48}{0.2}\right) = S_2\left(\frac{0.15}{0.2}\right)$ $S_1 = \frac{0.75S_2}{2.4}$ ...①

$2.4S_1 = 0.75S_2$

~~(↑) $S_1\left(\frac{0.52}{0.2}\right) = S_2\left(\frac{0.15}{0.}\right)$~~

(↑) $S_1\left(\frac{0.52}{0.2}\right) + S_2\left(\frac{0.25}{0.2}\right) = 21$

M1 A0

$2.6S_1 + 1.25S_2 = 21$ ...②

sub ① into ②,

$2.6\left(\frac{0.75S_2}{2.4}\right) + 1.25S_2 = 21$ ✓

$6.24(0.75S_2) + 3S_2 = 50.4$

$4.68S_2 + 3S_2 = 50.4$

$7.68S_2 = 50.4$

$S_2 = 6.56\text{N}$

From eqn ①,

$S_1 = \frac{0.75(6.56)}{2.4}$

$= 2.05\text{N}$

M1 A0

③

Total mark awarded = 3 out of 6

Examiner comment – 1 and 2

The majority of candidates attempted to resolve horizontally and vertically and then to solve the resulting simultaneous equations.

Both candidates have resolved using suitable angles as demonstrated by the link between the diagrams and equations. Candidate 1 made a slip when resolving vertically (writing 2 instead of 21), thus not gaining an accuracy mark. Candidate 2 used incorrect trigonometry to calculate the two angles between the vertical and the directions of the two strings. Although each angle is labelled the same (θ), it is clear from the diagrams and numerical work that, when resolving, the required angles are used. This gains the first two method marks, even though neither equation is numerically accurate.

Both candidates have made appropriate attempts to solve the simultaneous equations to obtain two different tensions. These gain the final method mark, although neither candidate could achieve accurate results from incorrect equations. Many candidates found the solution of the simultaneous equations challenging and a variety of errors occurred at this stage. Candidate 1 makes a commonly seen error when substituting for S_1, omitting to multiply by $\cos 67.38$. Candidate 2's solution shows another example of an error in manipulating the S_2 equation when attempting to multiply each side by 2.4.

Question 5

5 An object of mass 12 kg slides down a line of greatest slope of a smooth plane inclined at 10° to the horizontal. The object passes through points A and B with speeds $3\,\mathrm{m\,s^{-1}}$ and $7\,\mathrm{m\,s^{-1}}$ respectively.

(i) Find the increase in kinetic energy of the object as it moves from A to B. [2]

(ii) Hence find the distance AB, assuming there is no resisting force acting on the object. [3]

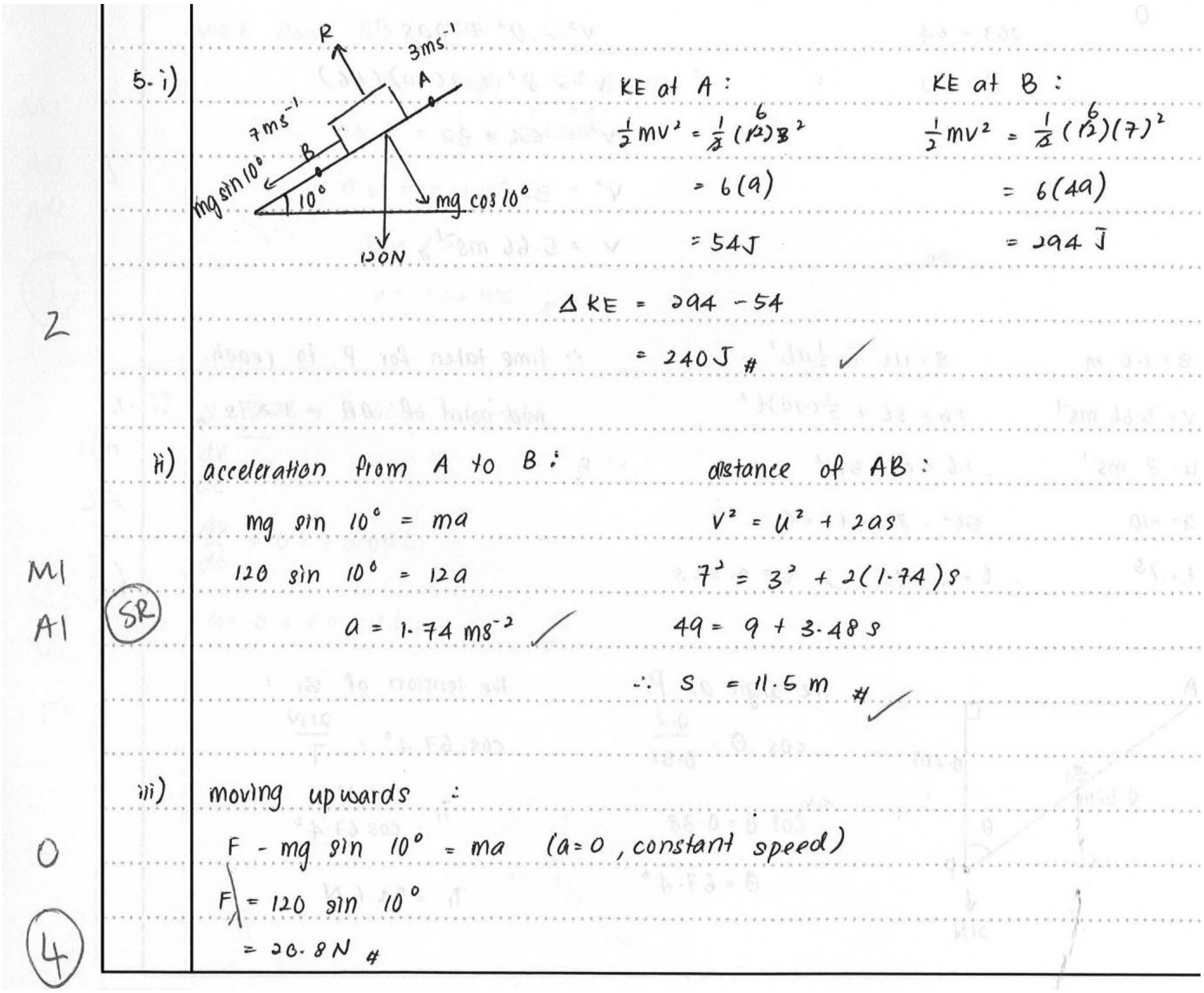
The object is now pushed up the plane from B to A, with constant speed, by a horizontal force.

(iii) Find the magnitude of this force. [3]

Mark scheme

5	**(i)**	$[\frac{1}{2}\,12(7^2 - 3^2)]$	M1		For using KE = $\frac{1}{2}m(v_B^2 - v_A^2)$
		Increase is 240 J	A1	2	
	(ii)		M1		For using mgh = KE gain
		$12g \times AB\sin 10^\circ = 240$	A1ft		
		Distance is 11.5 m	A1	3	
					SR for candidates who avoid 'hence' (max 2/3) For using Newton's Second Law and $v^2 = u^2 + 2as$ $[12g\sin 10^\circ = 12a$ $7^2 = 3^2 + 2(g\sin 10^\circ \times AB)]$ M1 11.5 m A1
	(iii)		M1		For using $F(AB)\cos 10^\circ$ = PE gain or for using Newton's 2nd law with $a = 0$.
		$F \times 11.5\cos 10^\circ = 240$ or $F\cos 10^\circ - 12g\sin 10^\circ = 0$	A1ft		
		Magnitude is 21.2 N	A1	3	

Example candidate response – 1



5. i) KE at A:

$\frac{1}{2}mv^2 = \frac{1}{2}(12)3^2$

$= 6(9)$

$= 54J$

KE at B:

$\frac{1}{2}mv^2 = \frac{1}{2}(12)(7)^2$

$= 6(49)$

$= 294 J$

$\Delta KE = 294 - 54$

$= 240J$ #

2

ii) acceleration from A to B:

$mg \sin 10° = ma$

$120 \sin 10° = 12a$ M1

$a = 1.74 ms^{-2}$ A1 SR

distance of AB:

$v^2 = u^2 + 2as$

$7^2 = 3^2 + 2(1.74)s$

$49 = 9 + 3.48s$

$\therefore s = 11.5 m$ #

iii) moving upwards:

$F - mg \sin 10° = ma$ ($a = 0$, constant speed)

$F = 120 \sin 10°$

$= 20.8 N$ #

0

4

Item marks awarded: (i) = 2/2; (ii) = 2/3; (iii) = 0/3

Total mark awarded = 4 out of 8

Example candidate response – 2

(5)

(i) Increase in kinetic energy $= \frac{1}{2}m(v-u)^2$

$= \frac{1}{2}(12)(7-3)^2$

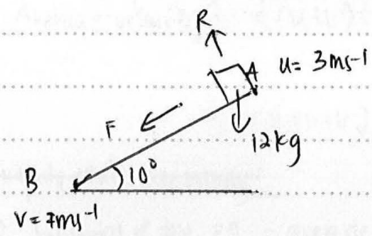
$= 6(4)^2$

~~$= 96$ ms⁻¹ J~~

$= 96$ J

0

(ii)



→ $F = ma$

$mg\sin\theta = ma$

$12\times10\times\sin 10° = 12a$ ✓

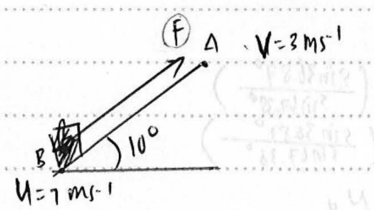
M1 (SR) $a = 1.74$ ms⁻²

A1 → $v^2 = u^2 + 2as$

$7^2 = 3^2 + 2(1.74)s$ ∴ the distance of AB is 11.5 m

$s = 11.5$ m ✓

(iii)



$mgh = \frac{1}{2}m(v-u)^2 +$ resistance

$12\times10\times11.5 = \frac{1}{2}(12)(7-3)^2 + F - 120\sin10°$

$mgh = \frac{1}{2}m(v-u)^2 + [F -$ resistance friction $]$

0 $12\times10\times11.5 = \frac{1}{2}\times12\times(7-3)^2 + [F - (12\times10\times\sin10°)]$

$1380 = 96 + F - 120\sin10°$

(2) $F = 1304.84$ N

Item marks awarded: (i) = 0/2; (ii) = 2/3; (iii) = 0/3

Total mark awarded = 2 out of 8

Examiner comment – 1 and 2

(i) Candidate 1 has calculated the difference between the two kinetic energies accurately to gain both marks. Candidate 2 has calculated $\frac{1}{2}m(v-u)^2$ instead of $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$.

(ii) The question asks candidates to 'hence' find the distance AB and so they need to use an energy method.

Both of these candidates, like many others, used an alternative method. They calculated the acceleration from $F = ma$ and then applied $v^2 = u^2 + 2as$ to obtain the distance AB. Both solutions show a correct distance for AB, but without using the kinetic energy increase found in part **(i)** the maximum marks available are 2 out of 3.

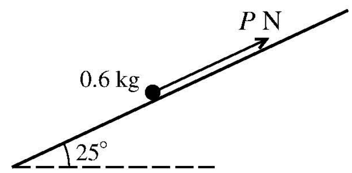
(iii) In common with many others, both of these candidates either misinterpreted or overlooked the word 'horizontal'. Both either show or imply that the force is directed up the inclined plane, instead of horizontally.

Candidate 1 used Newton's second law to obtain $20.8\,\text{N}$, an incorrect but frequently seen answer.

Candidate 2 has attempted to form an energy equation but has included too many terms, as well as including force and energy in the same equation. Candidates should check that the dimensions of each term in an equation are appropriate.

Question 6

6



The diagram shows a particle of mass 0.6 kg on a plane inclined at 25° to the horizontal. The particle is acted on by a force of magnitude P N directed up the plane parallel to a line of greatest slope. The coefficient of friction between the particle and the plane is 0.36. Given that the particle is in equilibrium, find the set of possible values of P. [9]

Mark scheme

6	$[P = \pm \mathrm{F} + 0.6g\sin 25^\circ]$	M1		For resolving forces in the direction of P
	$P_{max} = \mathrm{F} + 0.6g\sin 25^\circ$ or '$P = \mathrm{F} + 0.6g\sin 25^\circ$ when the particle is about to slide upwards'	A1		
	$P_{min} = -\mathrm{F} + 0.6g\sin 25^\circ$ or '$P = -\mathrm{F} + 0.6g\sin 25^\circ$ when the particle is about to slide downwards'	A1		
	$R = 0.6g\cos 25^\circ$	B1		
	$[\mathrm{F} = 0.36 \times 0.6g\cos 25^\circ]$	M1		For using $\mathrm{F} = \mu R$
	$[P_{max} = 0.36 \times 0.6g\cos 25^\circ + 0.6g\sin 25^\circ$, $P_{min} = -0.36 \times 0.6g\cos 25^\circ + 0.6g\sin 25^\circ]$	DM1		For substituting for F to obtain values of P_{max} and P_{min}
	$P_{max} = 4.49$, $P_{min} = 0.578$ (accept 0.58)	A1		Dependent on first M mark
		M1		For identifying range of value for equilibrium
	Set of values is $\{P;\ 0.578 \le P \le 4.49\}$	A1	9	AEF; Accept 0.58 instead of 0.578 and accept < instead of ≤

Example candidate response – 1

6. $\mu = 0.36$ $R = mg\cos\theta$ $f = \mu R$

$P = mg\sin\theta$ $= 6\cos 25$ $= 0.36(5.44)$ B1

$= 5.44$ ✓ $= 1.96$ ✓ M1

M1

$P = mg\sin 25 + 1.96$ A1

$= 4.50$ ✗ A0

M0

A0

M0

A0

(4)

Total mark awarded = 4 out of 9

Example candidate response – 2

	6. $a = g \sin\theta$
	$= 10 \sin 25°$ ✗
	$=$ ~~16.9~~ $4.23\ ms^{-2}$.
M0	
A0	$F = ma$
A0	$P - F_r - mg\sin\theta = ma$
B1	$P - F_r - 6\sin 25° =$ ~~6~~ $0.6(4.23)$ ✓
M1	$P - (\mu(mg\cos\theta)) = 5.07141941$
	$P = 5.071419141 + (0.36(6\cos 25°))$ ✓
M0	$P = 7.03\ N$
A0	
M0	
A0	$P = mg\sin\theta$ $\quad$ $P = mg\cos\theta$
	$= 2.54N$ $\quad$ $=$
(2)	

Total mark awarded = 2 out of 9

Examiner comment – 1 and 2

Many solutions to this question considered friction acting in one direction only, usually down the plane as in these examples. The question asked for a ‘set of possible values for P’. This should have suggested more than one or even two values for the frictional force. Candidates need to be clear about when to use $F = \mu R$ and when to use $F \leqslant \mu R$.

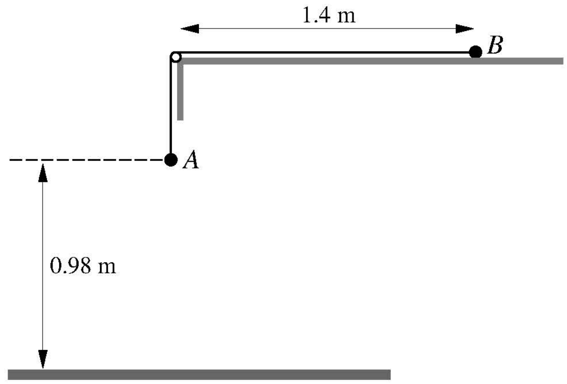
Both of these candidates find the normal reaction and then use it to obtain the frictional force for one situation, thus gaining two marks.

Candidate 1 has formed a correct equation for limiting equilibrium and solved this to obtain one value of P. Candidates should be aware that early rounding can lead to some loss of accuracy (in this case 4.50 instead of 4.49).

Candidate 2 has found the resultant force up the plane but has then equated this to a non-zero ma, thus gaining no further marks.

Question 7

7



Particles A and B have masses 0.32 kg and 0.48 kg respectively. The particles are attached to the ends of a light inextensible string which passes over a small smooth pulley fixed at the edge of a smooth horizontal table. Particle B is held at rest on the table at a distance of 1.4 m from the pulley. A hangs vertically below the pulley at a height of 0.98 m above the floor (see diagram). A, B, the string and the pulley are all in the same vertical plane. B is released and A moves downwards.

(i) Find the acceleration of A and the tension in the string. [5]

A hits the floor and B continues to move towards the pulley. Find the time taken, from the instant that B is released, for

(ii) A to reach the floor, [2]

(iii) B to reach the pulley. [3]

Mark scheme

7	**(i)**		M1		For applying Newton's 2^{nd} law to A or to B.
		$0.32g - T = 0.32a$ (or $T = 0.48a$)	A1		
		$T = 0.48a$ (or $0.32g - T = 0.32a$) OR $0.32g = (0.32 + 0.48)a$	B1		
			M1		For solving for a and T
		Acceleration is 4 ms^{-2} and tension is 1.92 N	A1	5	
	(ii)	[$0.98 = ½\, 4t^2$]	M1		For using $s = ½\, at^2$
		Time taken is 0.7 s	A1	2	
	(iii)		M1		For using v = at for taut stage and $t = d/v$ for slack stage
		$v = 4 \times 0.7$ and $t = (1.4 - 0.98)/v$ (= 0.15)	A1ft		ft a from **(i)** and /or t from **(ii)** ($a>0$, $a \neq g$)
	Time taken is 0.85 s		A1	3	

Example candidate response – 1

7. i) $T_1 = 0.48g\,a$ $\quad T_1 - mg = ma$ (1)

$T_1 =$ ~~4.8N~~ $0.48a$ ~~$T_1 =$~~ $T - 0.48(10) = 0.48a$ B1

$T - 4.8 = 0.4a$

$mg - T_2 = ma$

$(0.32)(10) - T_2 = 0.32a$ — (2)

~~$\frac{3.2 - 4.8}{0.32} = a$~~ $\quad 3.2 - 0.48a = 0.32a$ M1

$3.2 = 0.8a$ A1

~~$a =$~~ $\quad a = 4\ ms^{-2}$

M1

~~$T_1 = 0.48(4)$~~ $\quad T_2 = 3.2 - 0.48(4)$ A0

$= 1.28\ N$

(ii) $s = 0.98$, $t = ?$, $a = 4$, $u = 0\ ms^{-1}$

$s = ut + \frac{1}{2}at^2$

$0.98 = 0 + \frac{1}{2}(4)(t)^2$

$\frac{0.98}{2} = t^2$

$t = \sqrt{0.49}$ 2

$t = 0.7s$

(iii) $s = 1.4m$, $t = ?$, $a = 4$, $u = 0$

$s = ut + \frac{1}{2}at^2$ $\quad$ when ~~$t = 0.7s$; $s = \frac{1}{2}(u+v)t$~~ 0

$1.4 = 0 + \frac{1}{2}(4)t^2$ $\quad$ ~~$1.4 = \frac{1}{2}(0+v)\,0.7$~~

$\frac{1.4}{2} = t^2$ $\quad =$

$t = \sqrt{0.7}$ 6

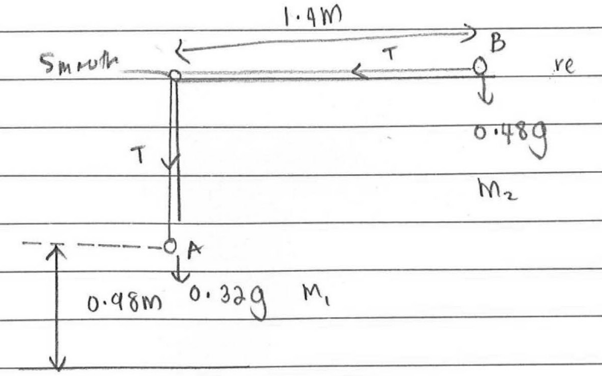
$t = 0.83\ s$

Item marks awarded: (i) = 4/5; (ii) = 2/2; (iii) = 0/3

Total mark awarded = 6 out of 10

Example candidate response – 2

7.



(i)
$M_1 g - T = M_1 a$ ①
$T - M_2 g = M_2 a$ ② — M1
$3.2 - T = 0.32a$ –① ✓ — A1
$T - 4.8 = 0.48a$ –② → $T = 0.48a + 4.8$ –③ — B0
Sub ③ into ①
$3.2 - 0.48a - 4.8 = 0.32a$
$3.2 - 4.8 = 0.32a + 0.48a$
$-1.6 = 0.8a$
$a = -2ms^{-2}$
∴ acceleration is $2ms^{-2}$

Sub $a = 2$ into ③
$T = 0.48(-2) + 4.8$ — M1
∴ $T = 3.84N$ — A0

(ii)
$S = ut + \frac{1}{2}at^2$
$0.98 = (0)t + \frac{1}{2}(2)t^2$ ✓ — M1
$t^2 = 0.98$ — A0
$t = 0.99s$

(iii)
$S = ut + \frac{1}{2}at^2$
$1.4m = 0t + \frac{1}{2}(2)t^2$
$t^2 = ~~1.48~~\ 1.4$ — 0
$t = 1.18s$

④

Item marks awarded: (i) = 3/5; (ii) = 1/2; (iii) = 0/3

Total mark awarded = 4 out of 10

Examiner comment – 1 and 2

(i) Both candidates have attempted to apply Newton's second law to the two particles and then to solve the resulting equations. Many candidates appeared to have memorised equations for a pulley system with both strings vertical but they were not always able to amend the equations for a different situation, such as the particle B attached to the horizontal string.

Candidate 1 obtained two correct equations and although the tensions are shown as T_1 and T_2, the solution of the equations to find the acceleration implies the use of $T_1 = T_2$. The acceleration is found accurately, but the substitution to find the tension in the string is incorrect.

Candidate 2 formed an incorrect pair of equations for a pulley system with both strings vertical, and solved them to obtain the incorrect values $a = -2\,\text{ms}^{-2}$ and $T = 3.84\,\text{N}$.

(ii) Both candidates have applied $s = ut + ½\,at^2$ to find the time taken for travelling the distance $0.98\,\text{m}$. Candidate 1 used the correct value $a = 4$ found in part **(i)** and calculated the time taken accurately.

Candidate 2, having obtained $a = -2$ in part **(i)** realised that the acceleration should be positive and used $a = 2$ but could not gain the accuracy mark for this part of the question.

(iii) Both of these candidates have applied the constant acceleration formula $s = ut + ½\,at^2$ with $s = 1.4\,\text{m}$ and the value for a found in part **(i)**. Although this was a common solution, to score any marks candidates needed to consider two stages of motion, the first with acceleration before particle A hits the floor and the second stage with no acceleration when the string becomes slack.

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