

MATHEMATICS

Paper 9709/11

Paper 11

Key Messages

- Attention needs to be paid to making sure workings are carried out at a sufficient level of accuracy to ensure the accuracy of the final answer.
- Candidates need to be careful to eliminate arithmetic and numerical errors in their answers.

General Comments

This paper covered the whole breadth of the syllabus and for those candidates who had covered the syllabus thoroughly and had practised by answering previous papers, it gave the opportunity to achieve high marks.

General setting out was mostly satisfactory, but again it is necessary to request that candidates do not place questions, or parts of questions, side by side on the page, as this can make it much more difficult to follow their working.

Comments on specific questions

Question 1

Most candidates were able to make a good attempt at the first part of this question on the binomial expansion. A few candidates gave the second, third and fourth terms, which of course also affected their answers to the last part of the question. A few candidates misunderstood the word 'ascending' and gave the seventh, sixth and fifth terms. Candidates were not quite so successful with part (ii) of this question.

Answers: (i) $64 + 576x + 2160x^2$; (ii) -3.75 .

Question 2

Most candidates realised that it was necessary to integrate the given function in order to find $f(x)$. For the first term, division by the new power ($\frac{1}{2}$) sometimes resulted in multiplication by $\frac{1}{2}$. Another common source of error was the omission of a constant of integration.

Answer: $2(x+6)^{1/2} - \frac{6}{x} - 3$.

Question 3

This was a straightforward vector question, and most candidates coped very well with it and obtained good marks.

Answers: (i) $\mathbf{DB} = 6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$, $\mathbf{DE} = 3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$; (ii) 17.2° .

Question 4

This was a common type of trigonometric equation requiring, as a first step in part (i), the use of the identity $\cos^2 x + \sin^2 x = 1$. A majority of candidates employed the correct procedure and were able to solve the resulting quadratic equation in $\cos x$ and hence to find the required two solutions in x . In part (ii), a surprising number of candidates did not appear to spot the connection between the two parts but proceeded to go through the whole process again. However, many candidates did realise that all that was required was

to equate $\frac{1}{2} \theta$ with each of the solutions from part (i), rejecting the larger of the resulting values of θ because it fell outside the required range.

Answers: (i) $60^\circ, 300^\circ$; (ii) 120° .

Question 5

The majority of candidates were able to reach the correct expression in part (i) for the inverse of the function, but very few candidates seemed to realise that they needed also to state the domain of the inverse function. In part (ii), most candidates were able to write down the correct expression for $f(f(x))$ and many were able to correctly start the process of solving the resulting equation. Unfortunately, instead of completing the square and taking the square root twice, many candidates expanded $(x^2 + 1)^2$ and thereafter mistakes were all too common.

Answers: (i) $f(x) = \sqrt{x-1}$ for $x \geq 1$; (ii) $x = 1.5$.

Question 6

Candidates usually made good attempts at writing down expressions for the perimeter in part (i), and the area in part (ii), but all too often made no attempt at simplifying the answers, which was specifically requested in the question. In part (iii), candidates were often able to obtain a method mark for using their answer to part (ii), but the correct answer eluded many candidates.

Answers: (i) $2\pi r + r\alpha + 2r$; (ii) $(3r^2\alpha)/2 + \pi r^2$; (iii) $\alpha = (2\pi)/5$.

Question 7

Part (i) was a standard type of question, asking candidates to find the perpendicular bisector of a line joining two given points, but candidates surprisingly did not take full advantage of the straightforward nature of this part. A significant proportion of candidates found the equation of AB first, which was not necessary, before going on to attempt the equation of the perpendicular bisector. Some candidates did not proceed beyond the equation of AB . Part (ii) was more challenging and depended on candidates realising that, in their answer to (i), they needed to replace (x, y) by (p, q) , as well as forming a second equation ($p^2 + q^2 = 4$), and solving the two equations simultaneously.

Answers: (i) $y = 2x - 2$; (ii) $(0, -2), (8/5, 6/5)$.

Question 8

Some candidates found obtaining the given answer in part (i) quite challenging, and struggled to obtain the given equation. Part (ii) was more successful in that most candidates were able to find the value of r corresponding to a stationary value of A and to demonstrate that A was a maximum. However, very few candidates substituted back to show that the corresponding value of x was zero and hence that there were no straight sections of the track.

Question 9

In part (a), the majority of the candidates wrote down correctly the equation resulting from the sum of the first 10 terms but a common mistake for the second equation was to equate $10(2a + 19d)$ to 1000 instead of to 1400. Hence many candidates lost marks in part (a). In contrast to this, part (b) was done quite well with most candidates obtaining the two equations correctly and going on to derive the correct quadratic equation. Candidates working in decimals had to be particularly careful in finding the value of a , having found the value of r to be 0.714(2). They needed to observe the rule that when the final answer is required correct to 3 significant figures, for example, calculations need to carry at least 4 significant figures. Hence candidates who used $r = 0.714$ when calculating a obtained an incorrect 3 significant figure answer of 1.72.

Answers: (a) $d = 6, a = 13$; (b) $r = 5/7$ (or 0.714), $a = 12/7$ (or 1.71).

Question 10

Part (i) was reasonably well done. A common mistake in finding the derivative was to forget to multiply by -2 , the derivative of the function inside the bracket. Similarly, in part (ii), a common mistake in integrating was to forget to divide by -2 . There were a number of different mistakes in part (ii) that were seen, many of them arithmetic. The outcome was that the correct answer was unfortunately not seen as regularly as anticipated.

Answers: (i) $y = -24x + 20$; (ii) $9/8$ (or 1.125).

MATHEMATICS

Paper 9709/12

Paper 12

Key Messages

Centres need to remind their candidates to read questions carefully, and of the need to make sure workings are carried out at a sufficient level of accuracy to ensure the accuracy of the final answer.

General Comments

It was pleasing that many candidates were able to show what they had been taught and what they had learnt and there were many very good scripts. However, there were some candidates for whom the paper was too challenging. There was very little evidence that candidates were short of time. **Questions 1** and **2** caused most candidates difficulty, but full marks were often seen on the last two questions. In **Question 6 part (ii)**, nearly half of the candidates did not read the question carefully enough and gave the stationary value of x , and not A as requested.

Comments on Specific Questions

Question 1

This was very badly answered, with the majority of candidates seemingly having not experienced this type of question. A small proportion of the candidates were able to use the identities $\sin^2 x + \cos^2 x = 1$ and $\frac{\sin x}{\cos x} = \tan x$ to solve parts (i) and (ii) but very few realised that $\tan(\frac{1}{2}\pi - x)$ was equal to the reciprocal of $\tan x$.

$$\text{Answer: (i) } \sqrt{1-p^2}, \text{ (ii) } \frac{\sqrt{1-p^2}}{p}, \text{ (iii) } \frac{p}{\sqrt{1-p^2}}.$$

Question 2

In part (i), most candidates did not realise firstly that the circumference of the base of the cone formed the arc length of the sector, and secondly that the slant height of the cone (10, found using Pythagoras' Theorem) formed the radius of the sector. Although the formulae " $s = r\theta$ " and " $A = \frac{1}{2}r^2\theta$ " were well known, these were rarely used with $r = 10$.

Answer: (i) 1.2π , (ii) 60π . (or decimal equivalents)

Question 3

This question was very well answered and most candidates achieved full marks. In part (i) omission of “×5” in the differentiation and in part (ii), omission of “÷5” in the integration were the most frequent errors. Also in part (ii), $2\div\frac{1}{2}$ was often equated to 1. A small minority of candidates, including some who scored highly, did not recognise that $\frac{1}{\sqrt{5x-6}}$ was $(5x-6)^{\frac{1}{2}}$.

Answers: (i) $-\frac{5}{6}$. (ii) $\frac{4}{5\sqrt{5x-6}}$, 0.8.

Question 4

- (i) Vector **AB** was almost always correct and it was particularly pleasing that the majority recognised that the unit vector in the direction of **AB** required division by 7 (the modulus of **AB**). Taking the modulus of $3\mathbf{i}-2\mathbf{j}+6\mathbf{k}$ as $\sqrt{(3^2 - 2^2 + 6^2)}$ was a common error and a significant proportion incorrectly took the question as being two-dimensional.
- (ii) Candidates were confident in their use of the scalar product apart from occasional errors when multiplying by 0. Dealing with $\cos^{-1}\frac{1}{5}$ caused considerable problems to at least a half of all candidates; these candidates did not realise that $\cos \theta = \frac{1}{5}$. The solution of $p^2 = 64$ was often given as $p = 8$, rather than $p = \pm 8$.

Answers: (i) $\frac{1}{7}(3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k})$. (ii) $p = \pm 8$.

Question 5

Most candidates correctly found the coordinates of *C* by finding the gradient of *BC* (perpendicular to *AB*) and then by solving the simultaneous equations for *AC* and *BC*. A significant number of solutions however were seen in which the diagonals of the rectangle *ABCD* were assumed to be at right angles. Methods for finding the coordinates of *D* were split more or less equally by a vector method (vectors **CD** and **BA** being equal), or by using the fact that the diagonals bisect or by using simultaneous equations with *BD* and *CD*.

Answer: (i) *C* (16, 6), *D* (12, 14).

Question 6

- (i) This question caused most candidates difficulty with only a small proportion of candidates being able to use similar triangles to express *y* in terms of *x*. Others chose to find the equation of the line *ST* and were more successful. Surprisingly, even when *y* was given in terms of *x*, many candidates did not realise that the area of the rectangle was *xy*. At least a quarter of all solutions attempted to express the area of the rectangle as “96 – the sum of the areas of triangles *SRQ* and *QPT*”. The candidates who did not equate this with “*xy*” were unable to make further progress.
- (ii) This was very well answered with the majority of candidates correctly setting the differential to 0 and deducing that the stationary value was a maximum. Unfortunately a large number of candidates did not read the question correctly and gave the stationary value of *x* instead of *A*.

Answers: (ii) *A* = 48 unit², maximum.

Question 7

- (a) (i) Use of the formula for the n th term was well done, but misuse of units (minutes and seconds) led to many incorrect answers.
- (ii) As in part (i), use of the formula for the sum of n terms was well known, but candidates again had difficulty with units.
- (b) This question was very well answered with most candidates scoring full marks. Finding $r = \frac{3}{2}$ instead of $\frac{2}{3}$ was a common error and one which should have been checked since the sum to infinity only exists for $|r| < 1$.

Answers: (a) (i) 21, (ii) 3 hours 15 minutes. (b) 216.

Question 8

- (i) Whilst most candidates realised that $x = \cos^{-1}(\frac{2}{3})$, answers given in radians or not to three significant figures, meant that many final answers were not correct.
- (ii) Only about a half of all solutions offered the end-points of -5 and 1 , and many of these used inequality signs incorrectly; $x < -5$ and $x < 1$ being a typical incorrect answer.
- (iii) The sketch graphs were generally well drawn, though many candidates wasted a lot of time in producing accurate graphs, rather than sketch graphs. Many examples were seen in which the graphs were either straight lines or parabolic in shape. It was rare to see the curve “flattening” at $x = 0$ and $x = 2\pi$.
- (iv) Only a small proportion of solutions to the inverse of the function were fully correct.
- (v) The responses to this part of the question were excellent. Apart from the occasional algebraic error and leaving the answer in terms of y rather than x , full marks were nearly always awarded.

Answers: (i) 0.841, 5.44. (ii) $-5 \leq f(x) \leq 1$. (iii) Graph. (iv) π . (v) $g^{-1}(x) = \cos^{-1} \frac{x+2}{3}$.

Question 9

This question proved to be a source of high marks.

- (i) The majority of candidates recognised the need to find the value of $\frac{dy}{dx}$ at $x = 1$ and multiply by 0.04 (the rate of change of x with respect to t).
- (ii) Apart from a few weaker candidates who attempted to integrate y , the vast majority of candidates realised the need to integrate πy^2 . Common errors however were to square $a + b$ as $a^2 + b^2$ or to integrate $\left(\frac{8}{x} + 2x\right)^2$ as $\frac{1}{3}\left(\frac{8}{x} + 2x\right)^2$. Use of the limits 2 to 5 was very good.

Answers: (i) -0.24 . (ii) 271.2π or equivalent decimal answer.

Question 10

This question was a source of high marks and many were completely correct.

- (i) The majority of candidates correctly solved the quadratic equation to obtain $x = 3$ and $-1\frac{1}{2}$, but again errors over the sign of the inequality led to a loss of the final accuracy mark. Such answers as $-1.5 < x < 3$ or $x > -1.5, x > 3$ were widespread.
- (ii) Although many candidates were confident with the process of completing the square, the “ $2x^2$ ” proved to be too difficult for many candidates. Whilst the value of a was almost always correct, b was often given as $-\frac{3}{2}$ instead of $-\frac{3}{4}$ and the values of $-\frac{9}{4}$ and $-\frac{9}{16}$ were seen as often as the correct value for c of $-\frac{9}{8}$. Recognising that the coordinates of the vertex were $(-b, c)$ seemed to be unfamiliar to many candidates.
- (iii) This part of the question was extremely well answered, with the vast majority of candidates correctly using “ $b^2 - 4ac = 0$ ” to obtain a correct value for k . Only a few solutions were seen in which either numerical slips or using “ $b^2 - 4ac < 0$ or > 0 ” affected the final answer.

Answers: (i) $x < -1\frac{1}{2}, x > 3$. (ii) $2\left(x - \frac{3}{4}\right)^2 - \frac{9}{8}, \left(\frac{3}{4}, -\frac{9}{8}\right)$. (iii) $k = \frac{27}{8}$.

MATHEMATICS

Paper 9709/13

Paper 13

Key Messages

Many candidates showed their working out which meant that wrong final answers could still receive credit for correct working. All candidates would benefit from following this very important advice, including for example when solving quadratic equations. Incorrect answers obtained from the use of a calculator with no working out were unfortunately unable to receive any credit. A better approach would be to: quote the formulae used; show values substituted into the formulae; and then write down the answer obtained by using a calculator.

General Comments

The paper seemed to be well received by the candidates and many good and excellent scripts were seen. The standard of presentation was generally good. The first 6 questions in the paper were reasonably straightforward, giving all candidates the opportunity to show what they had learned and understood. The second half of the paper (particularly **Questions 8, 9 and 10**) provided more of a challenge, even for able candidates. Most candidates appeared to have sufficient time to complete the paper.

It is worth pointing out that centres and candidates should be aware that generally a question with parts labelled **(i)**, **(ii)** and **(iii)** implies that there is a link between the parts as opposed to questions with parts labelled **(a)**, **(b)** and **(c)** where the link may not exist.

Comments on specific questions

Question 1

This was a good source of marks for most candidates. Students generally were able to factorise correctly and find the critical values. Students who drew a sketch of the quadratic were usually successful in finding the correct inequality. A number of candidates correctly wrote down $(x - 2)(x + 1) > 0$ but then unfortunately, incorrectly concluded, that $x > 2$ and $x > -1$.

Answer: $x < -1, x > 2$.

Question 2

The vast majority of candidates recognised the need for integration with a large proportion being successful. Sometimes errors occurred with the division by $-\frac{1}{2}$ or in forgetting to include the constant of integration, but most candidates dealt with the fractional power correctly and then substituted (4, 5) to find 'c'.

Answer: $f(x) = -2x^{-\frac{1}{2}} + x + 2$

Question 3

This was a relatively straightforward co-ordinate geometry question and produced a high proportion of entirely correct solutions. In part (i) the vast majority of candidates correctly used the perpendicular gradient of $-\frac{1}{2}$, although some confused parallel with perpendicular and used the same gradient. In part (ii) there were two correct methods used: finding the co-ordinates of the point C and then using the appropriate distance formula or finding the length of AB and then dividing this length by 2.

Answer: $y - 1 = -\frac{1}{2}(x - 3)$, $AC = 13$

Question 4

Most candidates were able to find **OD** and **CD** correctly, although a significant number seemed to lack confidence with vectors and some left answering this question to the end or did not attempt it. Better candidates often tended to use notation to show the route to get the required vectors, for example $\mathbf{OD} = \mathbf{OA} + \frac{1}{2}\mathbf{AB}$ and this should be encouraged. There were some sign errors and $\mathbf{OD} = -4\mathbf{i} + 3\mathbf{j}$ was common, coming from $\frac{1}{2}\mathbf{AB}$. Finding the length of a vector and the scalar product of their vectors was done very well. A few candidates carried out $\mathbf{DO} \cdot \mathbf{DC}$ demonstrating an understanding of the required directions, but most used $\mathbf{OD} \cdot \mathbf{CD}$. The vast majority of candidates had learnt the formula $\cos \theta = \frac{\mathbf{DO} \cdot \mathbf{DC}}{|\mathbf{DO}| |\mathbf{DC}|}$ but a small number could not find the correct values. Occasionally, candidates ignored the instruction in the question and attempted to find the angle by using trigonometry. Candidates need to be aware that this approach will receive no credit.

Answer: $4\mathbf{i} + 3\mathbf{j}$, $4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}$, 63.4

Question 5

Both parts of this question were usually done very well. The vast majority of candidates were able to correctly interpret the given information, form the required equations and then solve them successfully to obtain the correct answers. In part (a) a number of weaker candidates were unable to correctly make the step from $\frac{a}{1-r} = 8a$ to $r = \frac{7}{8}$ sometimes obtaining $\frac{5}{6}$. Others seemed to misunderstand the question and had $\frac{8a}{1-r}$ equal to something. This may indicate that more care needs to be taken to read the question carefully. Similarly in part (b), the main problem seemed to be correctly reading the given information. 197 was sometimes used as the fifteenth or fiftieth term rather than the 5th.

Answer: $\frac{7}{8}$, 14

Question 6

There were a very large number of completely correct solutions to both parts of this question. Again very many candidates were able to correctly interpret the given information, form the required equations and then solve them successfully to obtain the correct answers. In part (a), the only common error was failure to subtract the area of the small sector, when finding k . However, in part (b), 22 appeared regularly and 5α was occasionally absent from the shaded perimeter. Twice was only very rarely on the wrong side of the equation.

Answer: 3.84, $\frac{4}{3}$

Question 7

Most candidates understood what was needed in order to solve the equation in part (a) and found, or wrote down, $\sin \frac{\pi}{3}$ correctly, although some candidates worked in degrees and evaluated $\sin (3.142/3)^\circ$. A value of x was then almost always found but a significant number forgot the \pm sign when taking the square root. In part (ii) the vast majority of candidates correctly knew to consider $\sin^{-1} \frac{1}{2}$ but many had difficulty finding one or both of the required answers in the given range. Many obtained one correct solution but only the better candidates found both. Some did not realise that, having to solve for 2θ initially, they must also investigate possible solutions in the interval 2π to 4π . Other candidates did not appear to be comfortable working in radians (in terms of π) and worked in degrees or a mixture of degrees and radians. There were relatively few candidates who incorrectly stated that $\sin(2\theta + \frac{\pi}{3}) = \sin 2\theta + \sin \frac{\pi}{3}$ or who tried to use the formula for $\sin(A + B)$, which did not lead to a solution.

Answer: $\pm 1.366, \frac{\pi}{4}, \frac{11\pi}{12}$

Question 8

Parts (i) and (ii) were usually done successfully, using binomial expansions, but a correct strategy for answering part (iii) was seen relatively rarely. It should be noted that where a question asks for the coefficient of a particular term it is acceptable, although time consuming, to give the entire expansion but the required term should then be identified from the expansion. A number of candidates attempted the entire question without reference to the binomial theorem and attempted with varying degrees of success to expand $(x+3x^2)^4$ and/or $(x+3x^2)^5$ from first principles. Those who tried expanding $(1+x+3x^2)^5$ from first principles usually earned little credit and often devoted a considerable amount of time to it. There was a small minority of candidates who thought that no term in any of parts (i), (ii) or (iii) contained a term in x^8 . In part (i) many used the binomial expansion to obtain the correct term although some did conclude that $(3x^2)^4 = 3x^2$ and a few also incorrectly identified $6(x)^2(3x^2)^2$ as the term in x^8 . In part (ii) most candidates identified the correct term as $10(x)^2(3x^2)^3$ but a significant minority then incorrectly arrived at a final coefficient of 30. In part (iii) many candidates did not really attempt to use parts (i) and (ii) (with the hint that terms in x^8 come from the expansions in parts (i) and (ii)) but tried starting again and attempted to expand the 3-term bracket as it stood. The final part did provide a high level of discrimination and, generally, only the strongest candidates were able to produce correct responses although only a few did not make any attempt. Many candidates could not identify $(1+u)^5$ as the key building block with $u = x + 3x^2$. Sometimes those who did spot the intended method failed to relate this to the answers given in parts (i) and (ii) and several wasted considerable time and effort in redoing the first two parts of the question. A number using $(1+u)^5$ used the term in $(x+3x^2)^4$ but omitted the final term.

Answer: 81, 270, 675

Question 9

The vast majority of candidates knew the correct approach to this question: differentiate, set to 0, solve and then check the second derivative to decide if it is a maximum or minimum point. This was, however, an unusual question in that the function also contained the factor k^2 , where k was a constant. Unfortunately this caused problems for many candidates who differentiated this factor as $2k$. Another common mistake was to lose the derivative of the second term, x . This gave an equation with no real roots. Candidates who managed to avoid making either of the above mistakes should have obtained the equation $k^2 = (x+2)^2$. From here it was hoped that candidates would square root both sides. Those that did this very often forgot the \pm sign in front of k and so lost one of the two solutions. However, many ignored this simple approach and expanded the bracket and then attempted to use the formula for the solution of a quadratic equation – usually making errors on the way. Those who continued usually found the second derivative correctly unless they had made the earlier error with ‘ k ’ and then it often disappeared. Those with wrong values of ‘ x ’ obviously knew they had to check the sign of the second derivative but could not get an expression that was definitely +ve or –ve, although most made a decision for max/min. There were some very good solutions with clear expressions and working. There were a few candidates who had the correct values of ‘ x ’ but did not show a substitution and occasional errors in substituting, such as $(2+x)^3$ becoming $(k-2)^3$.

Answer: $-2 + k$: min, $-2 - k$: max

Question 10

Part (i) proved to be more difficult for candidates than anticipated. Most completed the square, usually correctly, or differentiated and set to 0, and thereby obtained the lowest value of c , but relatively few were able to state the range of f . In the question candidates were asked to give the range in terms of c but it was fairly common to see a numeric answer such as $y > 2$. In part (ii), very many candidates were competently able to form the equations from the given information and then continue with the process of solving simultaneous and quadratic equations to arrive at a correct solution. For some, obtaining the correct first equation proved to be more difficult than obtaining the correct second equation and reasonably often, algebraic errors were made in eliminating one of a or b or in solving the resulting quadratic equation. A few candidates, after forming correct equations in a , b and x then failed to use $x = 1$ and proceeded to try and solve the two equations still in terms of the three “unknowns”.

Answer: $a = 2, b = 1$

Question 11

This proved to be a successful question for many candidates. In part (i), many candidates were successfully able to use the chain rule to differentiate y and then substitute $x = 0$ to obtain the equation of the tangent.

Mistakes were sometimes made in differentiating the given function and answers such as $\frac{1}{2}(4x^3 + 4)^{-1/2}$ were not uncommon and a few confused the tangent with the normal.

In part (ii) many candidates equated the line and the curve and then squared both sides to obtain the given result but some did not make the steps clear. Students should be encouraged to show their methods clearly when the answer is given in the question. Most candidates simplified the given equation to the quartic equation $x^4 - x^2 = 0$ but often lost at least one of the 3 solutions to this either by forgetting the negative root, -1 , or by dividing by x and losing the answer 0 .

In part (iii) very many candidates recognised the requirement to integrate y^2 and squared the equation of the curve. $x^4 + 4x + 4$ was then usually integrated and 0 and -1 substituted to obtain the correct answer. Candidates should ensure that they apply the limits in the correct order and that when one of the limits is zero this is still substituted. Failure to do observe either of these points sometimes resulted in a negative result.

Answer: $y - 2 = x, 0, -1, 1, \frac{11\pi}{5}$

MATHEMATICS

Paper 9709/21

Paper 21

Key Messages

Centres should encourage candidates to check that they have given the answer in the correct form and also that they have completed the question.

General Comments

All candidates appeared to have sufficient time in which to complete the paper. There were questions in which it was common to see answers given to either an insufficient degree of accuracy, or in decimal forms rather than as exact expressions.

Comments on Specific Questions

Question 1

Most candidates were able to obtain the correct critical values by squaring each side of the given equation and solving the resulting quadratic equation. Very few correct ranges were seen.

Answer: $x < -2$, $x < -\frac{3}{2}$

Question 2

- (i) Most candidates showed correctly that the x -coordinate of P lies between the given points.
- (ii) Most candidates were able to show the given result either by manipulating the given result to obtain $x^4 + 2x - 9 = 0$ or by manipulating $x^4 + 2x - 9 = 0$ to obtain the given result.
- (iii) Most candidates were able to apply the iterative formula correctly and give the iteration to 4 decimal places, however many gave their final answer as 1.55 rather than the correct answer of 1.56.

Answer: (iii) 1.56

Question 3

While many candidates were able to differentiate the given equation correctly, many found the solution of the resulting quadratic in e^x equated to zero problematic. For those that did solve the resulting equation correctly, very few chose to give an exact form as required choosing instead to give a rounded decimal answer which was then used when determining the nature of the stationary points. This use of a rounded decimal answer was only penalised once.

Answer: Maximum when $x = 0$, minimum when $x = \ln 4$

Question 4

- (i) Apart from the occasional sign error or arithmetic slip, most candidates were able to obtain correct values for a and for b .
- (ii) Many candidates factorised the given polynomial, having obtained the correct quotient and then did not complete the solution by not solving $p(x) = 0$.

Answer: (i) $a = 1, b = -10$ (ii) $x = 1, 2, -4$

Question 5

- (i) Most candidates contrived to obtain the given answer by manipulating their often incorrect derivatives, thus losing method marks that could otherwise have been gained. Candidates should be advised not to use this practice.
- (ii) The result of $\theta = \frac{\pi}{6}$ was common but there were varying degrees of success in the calculation of the x and y coordinates

Answer: (ii) $(-1, 3)$

Question 6

- (a) (i) Few candidates realised that they needed to divide each term in the numerator by e^{2x} . Those candidates that did were often not successful in the ensuing integration of the exponential term.
 - (ii) Very few candidates recognised the need to use the appropriate double angle formula before attempting to integrate, so correct solutions were rare.
- (b) Many completely correct solutions were seen.

Answer: (a)(i) $x - 3e^{-2x} (+c)$ (ii) $\frac{3 \sin 2x}{4} + \frac{3x}{2} (+c)$ (b) 4.84

Question 7

- (i) This question was done well by most; the main errors were when candidates chose to give the value of R correct to 2 decimal places rather than as an exact value as required. Similarly, the value of α was often given correct to 1 decimal place rather than the 2 decimal places as required.
- (ii) Very few candidates attempted this part of the question and of those did, most chose to ignore the word 'Hence' and so attempted spurious and unsuccessful approaches. Candidates should be reminded that the word 'Hence' implies that work done in the previous part of the question is to be made use of.

Answer: (i) $R = \sqrt{10}, \alpha = 18.43^\circ$ (ii) $69.2^\circ, 163.8^\circ, 214.6^\circ, 343.8^\circ$

MATHEMATICS

Paper 9709/22

Paper 22

Key Messages

Centres should encourage candidates to check that they have given the answer in the correct form and also that they have completed the question.

General Comments

Questions 1 and **2** were problematic for quite a few candidates, but subsequent questions were more accessible to most, apart from **Question 6** and parts of **Question 7**. Provided candidates were well prepared, the paper enabled candidates to show their mathematical strengths and understanding of the syllabus content. Candidates appeared to have adequate time to complete the paper.

Comments on Specific Questions

Question 1

- (i) If a candidate did not recognise the form of the integration needed, then marks were unable to be awarded. It was hoped that candidates may recall the necessary result or form of result when it came to looking at part (ii), but in many cases this did not appear to happen. Of those candidates who realised that the answer contained $\ln(4x - 1)$, many obtained an incorrect multiple of $\ln(4x - 1)$
- (ii) Most candidates were able to gain a mark for a correct attempt to use limits and their result from part (i). However many candidates contrived to obtain a result in the given form. Very few correct answers were seen, with errors of the type $\frac{1}{2}\ln 27 - \frac{1}{2}\ln 3 = \frac{\ln 27}{\ln 3} = \ln 9$ being common.

Answer: (i) $\frac{1}{2}\ln(4x - 1) (+ c)$, (ii) $\ln 3$

Question 2

The most important part of this question depended upon whether a candidate recognised that differentiation of either a product or a quotient was necessary. Too many candidates did not do this and as a result it was very difficult to award marks, as an equation of the form $e^{3x-1} = 0$ usually resulted. Many candidates did not realise that this could not be solved and offered many spurious solutions, the most common of which was $x = \frac{1}{3}$, unfortunately fortuitous. Candidates had to provide correct working in order to gain the marks for this common answer.

Answer: $\left(\frac{1}{3}, \frac{3}{2}\right)$

Question 3

Provided the correct identity was used, most candidates were able to make a good attempt at the resulting quadratic equation, with many correct solutions being seen.

Answer: $14.5^\circ, 165.5^\circ, 221.8^\circ, 318.2^\circ$

Question 4

- (i) Apart from the occasional sign error or arithmetic slip, most candidates were able to obtain correct values for a and for b .
- (b) Many candidates solved $p(x)=0$, but usually managed to obtain the marks available provided correct factors had been seen, hence the need for candidates to ensure that they are actually providing what is required as an answer.

Answer: (i) $a = 4, b = -3$, (ii) $(4x + 1)(x + 2)(x - 3)$

Question 5

- (i) Most candidates were able to make a good, correct attempt at obtaining $\frac{dy}{dx}$, with the occasional slip in obtaining $\frac{dx}{dt}$ or $\frac{dy}{dt}$. However, many candidates left the gradient in terms of t rather than finding the value of t when $y = 6$. Of those candidates that attempted to find the value of t , many did not give an exact answer as required, again highlighting the need for candidates to ensure that they check that they have given the answer in the required form.
- (ii) This part of the question was misunderstood by most candidates, with very few correct solutions being seen. Many candidates found the equation of the straight line, using their gradient from part (i) which passed through the point $(1,0)$, which would have been acceptable provided a check that the line passed through the point P had been made. This was very rarely done. It had been expected that candidates find the equation of the tangent at the point P and verify that this tangent passed through $(1,0)$. In fact, this approach was rarely used.

Answer: (i) $\frac{6}{e}$

Question 6

- (a) Most candidates attempted to expand out the brackets before attempting to integrate with respect to x . There were too many examples of candidates 'forgetting' $-2\sin x \cos x$. Many also did not use the fact that $\sin^2 x + \cos^2 x = 1$. Very few correct uses of the appropriate double angle formulae were seen.
- (i) There were very few completely correct solutions seen, due mainly to candidates having their calculator in the wrong mode, making errors in h or a complete misunderstanding of the function $\operatorname{cosec} x$.
- (ii) Very few candidates were able to give a reasonable sketch of the graph of $y = \operatorname{cosec} x$ over the given range. This was another example of candidates not ensuring that they had answered the question as required. Even fewer candidates made use of this graph, as requested, to explain whether the answer obtained in part (i) was an over-estimate or an under-estimate. It was hoped that candidates would show that the answer from part (i) was an over estimate by drawing in relevant trapezia.

Answer: (a) $x + \frac{\cos 2x}{2} (+c)$ (b)(i) 0.899 (ii) over-estimate

Question 7

- (i) This part of the question was not done well by many candidates. Some chose to differentiate rather than integrate the given function. Those that did integrate often applied the limits incorrectly and then contrived to obtain the given answer. Candidates should be encouraged to check their work when they are unable to obtain a given answer, rather than incorrectly manipulate or blatantly change terms in order to write down what appears to be correct. Candidates are more likely to gain method marks if they do not manipulate their incorrect results.
- (ii) Very few correct approaches were seen, with the majority of candidates not considering the sign of either $\sqrt{\left(\frac{2-e^a}{8}\right)} - a$ or $a - \sqrt{\left(\frac{2-e^a}{8}\right)}$ when $a = 0.2$ and $a = 0.3$. Most just considered the sign of $\sqrt{\left(\frac{2-e^a}{8}\right)}$ when $a = 0.2$ and $a = 0.3$ and then were unable to make a correct deduction.
- (iii) Most candidates were able to obtain full marks for this part, having adopted a completely correct method of solution throughout.

Answer: (iii) 0.29

MATHEMATICS

Paper 9709/23

Paper 23

Key Messages

Centres should encourage candidates to check that they have given the answer in the correct form and also that they have completed the question.

General Comments

All candidates appeared to have sufficient time in which to complete the paper. There were questions in which it was common to see answers given to either an insufficient degree of accuracy, or in decimal forms rather than as exact expressions.

Comments on Specific Questions

Question 1

Most candidates were able to obtain the correct critical values by squaring each side of the given equation and solving the resulting quadratic equation. Very few correct ranges were seen.

Answer: $x < -2$, $x < -\frac{3}{2}$

Question 2

- (i) Most candidates showed correctly that the x -coordinate of P lies between the given points.
- (ii) Most candidates were able to show the given result either by manipulating the given result to obtain $x^4 + 2x - 9 = 0$ or by manipulating $x^4 + 2x - 9 = 0$ to obtain the given result.
- (iii) Most candidates were able to apply the iterative formula correctly and give the iteration to 4 decimal places, however many gave their final answer as 1.55 rather than the correct answer of 1.56.

Answer: (iii) 1.56

Question 3

While many candidates were able to differentiate the given equation correctly, many found the solution of the resulting quadratic in e^x equated to zero problematic. For those that did solve the resulting equation correctly, very few chose to give an exact form as required choosing instead to give a rounded decimal answer which was then used when determining the nature of the stationary points. This use of a rounded decimal answer was only penalised once.

Answer: Maximum when $x = 0$, minimum when $x = \ln 4$

Question 4

- (i) Apart from the occasional sign error or arithmetic slip, most candidates were able to obtain correct values for a and for b .
- (ii) Many candidates factorised the given polynomial, having obtained the correct quotient and then did not complete the solution by not solving $p(x) = 0$.

Answer: (i) $a = 1, b = -10$ (ii) $x = 1, 2, -4$

Question 5

- (i) Most candidates contrived to obtain the given answer by manipulating their often incorrect derivatives, thus losing method marks that could otherwise have been gained. Candidates should be advised not to use this practice.
- (ii) The result of $\theta = \frac{\pi}{6}$ was common but there were varying degrees of success in the calculation of the x and y coordinates

Answer: (ii) $(-1, 3)$

Question 6

- (a) (i) Few candidates realised that they needed to divide each term in the numerator by e^{2x} . Those candidates that did were often not successful in the ensuing integration of the exponential term.
 - (ii) Very few candidates recognised the need to use the appropriate double angle formula before attempting to integrate, so correct solutions were rare.
- (b) Many completely correct solutions were seen.

Answer: (a)(i) $x - 3e^{-2x} (+c)$ (ii) $\frac{3\sin 2x}{4} + \frac{3x}{2} (+c)$ (b) 4.84

Question 7

- (i) This question was done well by most; the main errors were when candidates chose to give the value of R correct to 2 decimal places rather than as an exact value as required. Similarly, the value of α was often given correct to 1 decimal place rather than the 2 decimal places as required.
- (ii) Very few candidates attempted this part of the question and of those did, most chose to ignore the word 'Hence' and so attempted spurious and unsuccessful approaches. Candidates should be reminded that the word 'Hence' implies that work done in the previous part of the question is to be made use of.

Answer: (i) $R = \sqrt{10}, \alpha = 18.43^\circ$ (ii) $69.2^\circ, 163.8^\circ, 214.6^\circ, 343.8^\circ$

MATHEMATICS

Paper 9709/31

Paper 31

General comments

The standard of work on this paper varied considerably and resulted in a wide spread of marks from zero to full marks. Most questions discriminated well. The questions or parts of questions that were generally done well were **Question 2** (modular indicial equation), **Question 5 (i)** (trigonometry), **Question 7** (partial fractions) and **Question 9 (i)** (vector geometry). Those that were done least well were **Question 6** (geometry and iteration), **Question 8** (complex numbers), **Question 9 (iii)** (vector geometry), and **Question 10** (differential equation).

In general the presentation of work was good, though there were some untidy scripts. Candidates working towards a given answer as in **Question 5 (ii)** usually took care to provide the essential working. Two of the first questions in the paper, **Question 1** and **Question 4**, began by testing a standard method but then needed a simple imaginative step to complete the solution. Some candidates seemed to spend a considerable amount of time trying to finish these questions and this may have contributed to the impression that candidates did not have always enough time to attempt all the questions on this paper. Candidates need to be advised that it is important to ration the time spent on questions early in a paper to guard against difficulties later on.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on Specific Questions

Question 1

The initial differentiation was generally very well done and most candidates reached the simplified form $-1/(1+2x)^2$. In simplifying the numerator obtained by application of the quotient rule, the error of taking $-2(1+x)$ to be $-2+2x$ was quite common. The final mark, for explaining why the derivative is always negative, was easily earned by those who pointed out that $(1+2x)$, or its square, was always positive. However the majority of candidates wasted valuable time and earned nothing by calculating the gradient for several values of x or, in some cases, examining the second derivative.

Question 2

This was well answered by many candidates. Roughly equal numbers chose to either square the modular equation or consider a pair of non-modular linear equations. However mistakes in handling the modulus, indices and logarithms were common. For example errors such as taking $|3^x - 1|$ to be $\sqrt{(3^x - 1)}$, $2(3^x)^2$ to be 6^x , $(3^x)^2$ to be 9^{2x} and $\ln(3^x - 1)$ to be $x \ln 3 - \ln 1$ were frequently seen. Candidates generally need to be more secure in their manipulation of moduli, indices and logarithms.

Answer: 0.631, -0.369

Question 3

Most attempts at integrating by parts began well. The second stage usually involved the integral of an expression of the form $k\sqrt{x} \cdot \frac{1}{x}$. Weakness in handling indices let down some of those who tried to simplify such an expression. Others made the error of taking the integral of this product of terms to be equal to the product of their integrals. However there were many correct solutions.

Answer: $8 \ln 2 - 4$

Question 4

The method of parametric differentiation appeared to be well understood and $\frac{dy}{dx}$ many candidates obtained a correct expression for —and simplified it to say $\frac{\sin t - \cos t}{\cos t + \sin t}$. Since the given answer involved the tangent function the introduction of $\tan t$ into the fraction would seem a natural way forward, but very few attempts were seen to use this strategy. Multiplication of the numerator and denominator by $\cos t - \sin t$ or by $\cos t + \sin t$ was often seen and usually led to much fruitless work.

Question 5

Part (i) was generally well answered by a variety of methods. Candidates did not always realise that the identity given in part (i) could be used to assist the solution of part (ii). Those who saw the connection sometimes omitted to introduce a factor of $\frac{1}{2}$ but the ensuing integrations of $\cot \theta$ and $\tan \theta$ were usually done well. Moreover candidates went to considerable lengths to show sufficient working with logarithms to justify the given answer. Of those who did not use the result of part (i), the majority produced no work of value. However a minority did make progress by using the known formulae for the integral of cosec x , i.e. $\tan\left(\frac{1}{2}x\right)$ and $-\ln(\operatorname{cosec} x + \tan x)$.

Answer: Another successful approach was to express $\operatorname{cosec} 2\theta$ as $\frac{\sec^2 \theta}{2 \tan \theta}$ before integrating.

Question 6

Part (i) was poorly answered and very few correctly argued proofs were seen. It was common to see standard formulae for the areas of sectors and triangles written down with little or no attempt made to apply them to the angles and lengths actually involved in the question. The area of the shaded area was usually found by adding the area of the sector ABC with vertex at A to the minor segments on AB and AC . In part (ii) the first iteration was usually correctly calculated but after that the iterations were quite often wrong because a persistent error was made in applying the iterative formula. It was encouraging to see that nearly all candidates had set their calculators in radian mode for the calculations.

Answer: (ii) 0.95

Question 7

This was very well answered. In part (i) most candidates sought the correct form of partial fractions and had a sound method for finding the constants. In part (ii) those intending to use the standard expansion of $(1 + x)^n$ sometimes made errors in converting $(x - 2)^{-1}$ and $(x^2 + 3)^{-1}$ to $-\frac{1}{2}\left(1 - \frac{x}{2}\right)^{-1}$ and $\frac{1}{3}\left(1 + \frac{x^2}{3}\right)^{-1}$ respectively. In situations such as this the use of the standard expansion of $(a + b)^n$ is perhaps worth considering.

Answer: (i) $-\frac{1}{x-2} + \frac{3x-1}{x^2+3}$; (ii) $\frac{1}{6} + \frac{5}{4}x + \frac{17}{72}x^2$

Question 8

Part (a) was fairly well answered. The most popular approach was to eliminate u or v , obtain an interim solution such as $v = \frac{5}{1-2i}$ and convert it to the required form.

However quite a few stopped at the interim answer and appeared to not know how to continue. Another method was to substitute $u = a + ib$ and $v = c + id$ in the given equations and derive four equations in a , b , c and d by equating real and imaginary parts. These equations were easily solved to give u and v in the required form. Some candidates set both u and v equal to $x + iy$ and scored nothing. In part (b) the diagrams were not often fully correct. The question requires no more than a sketch. Nevertheless it would be helpful if candidates (a) had axes with equal scales in mind, and (b) gave some indication of magnitude by for example marking 1 on the real axis and i on the imaginary axis. Most showed a circle of radius 1, and many had the centre at $-i$, though 1, -1 , i were also seen regularly. The half line was rarely correct. It often had the wrong starting point, or the wrong angle of inclination or was poorly drawn. Of those with a correct diagram very few had a sound method for finding the least value of $|z - w|$.

Answers: (a) $u = -2 - 2i$, $v = 1 + 2i$ (b) $\frac{3}{\sqrt{2}} - 1$

Question 9

Part (i) was well done on the whole. The most common method found a normal to the required plane by calculating the product of a relevant pair of vectors. The next most common approach obtained two linear equations in the components a , b and c of the normal by equating relevant scalar products to zero. A third and much less frequent method began by using relevant vectors to set up a two-parameter equation for the plane. Elimination of the parameters then gave the required Cartesian equation.

Part (ii) was fairly well done but the impression gained was that this was a challenging task for most candidates. Solutions were lengthy and inevitably prone to slips in calculation. The use of a standard and economic procedure for ratio division such as $\overrightarrow{OD} = \frac{2}{3}\overrightarrow{OB} + \frac{1}{3}\overrightarrow{OC}$ was hardly ever seen and needs to be encouraged.

A few candidates solved part (iii) by a wide variety of methods including the use of the vector product of \overrightarrow{OA} and \overrightarrow{OD} , often aiding their work with a simple diagram involving the relevant points and lines. But with most of the other attempts the lack of a diagram made the work difficult to follow. Expressions and equations were written down without clearly saying what they were meant to represent. Many solutions attempted to find the wrong distance (often the perpendicular from A to BC).

Answer: (i) $3x + y - 2z = 1$; (ii) $i + 2j + 2k$

Question 10

- (i) Most candidates found this to be a difficult problem. Often the only mark awarded was for stating or implying that $\frac{dV}{dt} = -k\sqrt{h}$. Some realised that they would need $\frac{dV}{dh}$ but many of them failed to allow for the fact that r was variable and that $V dh$ had to be expressed in terms of h alone before finding the required derivative. Those that attempted to express r in terms of h often made an elementary trigonometric error.
- (ii) Separation of variables and the subsequent integration was well done by many. Most of these candidates went on to gain at least one more mark by using either $h = 0$ when $t = 60$, or less frequently $h = H$ when $t = 0$. But very few used both boundary conditions even though the presence of two constants demanded them.
- (iii) There were very few correct solutions.

Answer: (ii) $t = 60\left(1 - \left(\frac{h}{H}\right)^{\frac{5}{2}}\right)$ (iii) 49.4

MATHEMATICS

Paper 9709/32

Paper 32

General comments

The standard of work on this paper varied; most questions discriminated well. The questions or parts of questions that were generally done well were **Question 2** (modular indicial equation), **Question 5 (i)** (trigonometry), **Question 7** (partial fractions) and **Question 9 (i)** (vector geometry). Those that were done least well were **Question 6** (geometry and iteration), **Question 8** (complex numbers), **Question 9 (iii)** (vector geometry), and **Question 10** (differential equation).

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Part (i) was well done on the whole. The most common method found a normal to the required plane by calculating the product of a relevant pair of vectors. The next most common approach obtained two linear equations in the components a , b and c of the normal by equating relevant scalar products to zero. A third and much less frequent method began by using relevant vectors to set up a two-parameter equation for the plane. Elimination of the parameters then gave the required Cartesian equation.

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A few candidates solved part (iii) by a wide variety of methods including the use of the vector product of \overrightarrow{OA} and \overrightarrow{OD} , often aiding their work with a simple diagram involving the relevant points and lines. But with most of the other attempts, the lack of a diagram made the work difficult to follow. Expressions and equations were written down without clearly saying what they were meant to represent. Many solutions attempted to find the wrong distance (often the perpendicular from A to BC).

Answer: (i) $3x + y - 2z = 1$; (ii) $i + 2j + 2k$

Question 10

- (i) Most candidates found this to be a difficult problem. Often the only mark awarded was for stating or implying that $\frac{dV}{dt} = -k\sqrt{h}$. Some candidates realised that they would need $\frac{dV}{dh}$ but many of them did not allow for the fact that r was variable and that V had to be expressed in terms of h alone before finding the required derivative. Those that attempted to express r in terms of h often made an elementary trigonometric error.
- (ii) Separation of variables and the subsequent integration was well done by many. Most of these candidates went on to gain at least one more mark by using either $h = 0$ when $t = 60$, or less frequently $h = H$ when $t = 0$. But very few used both boundary conditions even though the presence of two constants demanded them.
- (iii) There were very few correct solutions.

Answer: (ii) $t = 60\left(1 - \left(\frac{h}{H}\right)^{\frac{5}{2}}\right)$ (iii) 49.4

MATHEMATICS

Paper 9709/33

Paper 33

Key Messages

- A considerable proportion of the total marks in this paper depend on giving answers which are correct to 3 significant figures. In order to gain as many marks as possible all working should be either exact or correct to at least 4 significant figures.
- It is important for candidates to read the questions carefully, and answer as required.

General comments

Many candidates found this paper straightforward. The more challenging aspects of some questions, most notably **Question 9** (complex numbers) and **Question 10** (solving a differential equation), discriminated well, while the earlier questions allowed all candidates an opportunity to demonstrate their skills. The questions that were done well were **Question 1** (logarithmic equation), **Question 5** (Integration by parts and iteration), **Question 6** (vector equations of planes and lines) and **Question 8** (partial fractions). The work on integration by substitution (**Question 2**) was good in comparison with recent past papers, but candidates were less successful with the trigonometry (**Question 7**) than usual. Expression of a complex number in the form $re^{i\theta}$ (**Question 9**) and the integration of $\cos^2 x$ with respect to x (**Question 10**) were done least well.

Most candidates offered solutions to all ten questions, and a lot of very clear, accurate and concise scripts were seen. Candidates should be advised to avoid working in pencil, which often results in solutions that are difficult to read. Similarly, scripts where candidates work in two columns per page tend to be more difficult to follow than those where the candidate uses the full width of the page for their solution. It is important to check that the solution offered responds to the question; several candidates lost marks at the end **Question 1** because their answer gave a in terms of x rather than x in terms of a , and at the end of **Question 3** (factors and roots) because their answer referred to factors when the question asked about the roots. If an answer is given in the question (such as the trigonometric equation in **Question 7**) candidates should be aware that detailed working is required to score full marks.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only "correct answer".

Comments on Specific Questions

Question 1

This question was well done by many candidates, with only a minority demonstrating no understanding of the laws of logarithms. Most candidates achieved a correct equation without logarithms involved. In the work that followed there were some algebraic slips in rearranging to find x in terms of a , usually due to errors in expanding $(x+a)^2$. Some rearrangements resulted in an expression for x in terms of x and a . Some candidates did not reach the required final answer, leaving their solution as an expression for a in terms of x .

Answer: $\frac{16}{a-8}$

Question 2

This was a comparatively straightforward substitution, with many candidates reaching the correct first stage of $\int \frac{u-1}{3u} du$. Most errors at this stage were due to incomplete substitutions, most commonly resulting in an attempt to integrate a function of u with respect to x . Only a small proportion of the candidates who obtained the correct form recognised that this was very straight forward to integrate by rearranging it as $\int \frac{1}{3} - \frac{1}{3u} du$. Many attempted to use integration by parts, but only a few were successful. Candidates were expected to give their final answer in terms of x .

Answer: $\frac{1}{3}(3x+1) - \frac{1}{3}\ln|3x+1| + c$

Question 3

- (i) The most common, and most successful, method was to use the factor theorem by substituting $x = -2$ and equating to zero. Attempts to use long division or synthetic division often resulted in algebraic slips.
- (ii) Many candidates obtained the correct quadratic factor, and most of these went on to show that this factor has complex roots. In completing the response there was often no mention at all of the real root from the factor $(x+2)$. Several solutions indicated confusion between factors of an expression and roots of an equation. Given that there is a given result and three marks available for this part of the question, candidates should have realised that using a calculator to find the three roots of the equation was not an acceptable alternative.

Answer: (i) $a = -1$

Question 4

This question was answered well by many candidates. A small number of candidates did not use the correct rule for differentiation of a product and some left out the $\frac{dy}{dx}$ in the implicit differentiation. The most common error was to use $\frac{d}{dx}(14) = 14$. Marks were also lost due to losing terms in rearranging to form an expression for $\frac{dy}{dx}$ and arithmetic errors in substituting $x = 0$ and $y = 2$.

Answer: $-\frac{4}{3}$

Question 5

- (i) Most candidates made very good attempts at the integration by parts. There were some errors in the coefficients caused by errors in dividing by $\frac{1}{2}$ but, more worryingly, some candidates thought that the power of e would change, resulting in answers involving $e^{\frac{1}{2}x}$ and $e^{\frac{3}{2}x}$. Candidates who integrated correctly usually used the limits correctly to obtain the given result, but some did not give sufficient working to show how they had arrived at the given result, and some just could not see how to change the exponential form to the given logarithmic form.

- (ii) Most candidates worked through the iterative process correctly. Several candidates lost marks through not working to the accuracy required in the question, possibly due to confusion between decimal places and significant figures. A small number of candidates did not understand what was required, and substituted 3.5, 3.6, 3.7 ... etc., rather than use the outcome of one stage as the value to substitute in the next.

Answer: (ii) 3.77

Question 6

- (i) Most candidates understood how to use the scalar product to find the angle between two vectors. Some ignored the request for the acute angle, and some went wrong by using \sin in place of \cos , or subtracting 90° from their answer to obtain an answer in the required range. Candidates who made an error in evaluating the scalar product could only gain credit for a correct method if they showed sufficient working to demonstrate that they had used the correct method to obtain their value.
- (ii) Candidates used several different approaches to find the equation of the required line. The vector product was often used correctly to find the direction of the line, but the method of finding two points and then obtaining the direction was equally popular. All approaches were prone to arithmetic errors, and some found the equation of a plane perpendicular to the required line rather than the equation of the line itself. Some candidates who found a correct point then expressed it incorrectly in vector form so, for example, $(0, -17, -4)$ became $-4\mathbf{j} - 17\mathbf{k}$.

Answer: (i) 67.8° , (ii) $\mathbf{r} = (2\mathbf{i} - 3\mathbf{j}) + \lambda(\mathbf{i} + 7\mathbf{j} + 2\mathbf{k})$

Question 7

- (i) Many candidates either showed insufficient working or could not demonstrate the equivalence of the two given expressions. Most showed the substitutions $\sec \theta = \frac{1}{\cos \theta}$ and $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$, and many showed the expansion of $\sin 2\theta$, but the final step in the working to obtain the given result was often missing.
- (ii) There were many fully correct answers to this part of the question. Many candidates used a remembered form of the answer rather than working from first principles. This can be successful, but it can lead to incorrect statements such as $\sin \alpha = 4$ and $\cos \alpha = 2$ and to some confusion between α and θ . The most common error was the use of $\tan^{-1} \frac{1}{2}$ in place of $\tan^{-1} 2$.
- (iii) Most candidates spotted the link between this part of the question and parts (i) and (ii) and started by rewriting the equation in the form $R \sin(\theta + \alpha) = 3$. Most candidates who found $\sin^{-1}\left(\frac{3}{\sqrt{20}}\right)$ correctly went on to find a correct value for θ , but solutions giving two correct values and no incorrect values were unusual. Some candidates stopped when they had found one solution, and others were not confident in using the properties of the sine function to find additional solutions. Other candidates went wrong by using $\sin(\theta - 63.43)$ or through rounding errors.

Answer: (ii) $R = \sqrt{20}$, $\alpha = 63.43^\circ$ (iii) 74.4° , 338.7°

Question 8

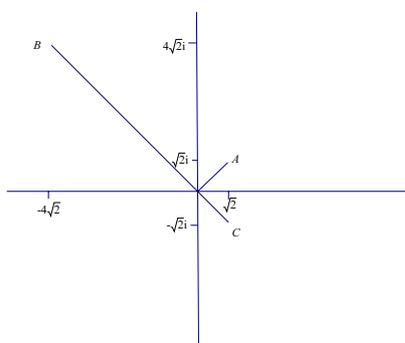
- (i) Most candidates made a confident attempt to split the fraction into an appropriate form using partial fractions. Fully correct answers to this part were common. Most errors were due to expressions with missing or additional terms, and there was some confusion between $(1+x)^2$ and $(1+x^2)$.
- (ii) A few candidates chose to work with the original form of the fraction in this part, but most went on to use their answer from part (i). The expansions of $(1+x)^{-1}$ and $(1+x)^{-2}$ were usually correct in their unsimplified forms, but $(2-3x)^{-1}$ often became $2\left(1-\frac{3}{2}x\right)^{-1}$. There were many correct final answers. Arithmetic slips and sign errors caused some solutions to go wrong in the final stages. Some candidates whose solution was originally correct then chose to double their answer to give an incorrect final answer with integer coefficients.

Answer: (i) $\frac{-1}{1+x} + \frac{3}{(1+x)^2} + \frac{4}{2-3x}$ (ii) $4 - 2x + \frac{25}{2}x^2$

Question 9

- (i) Despite the instruction to use the formula for the solution of a quadratic equation, several candidates could not find a way to start this question. Rather than use the method requested, some candidates chose to substitute $z = a + bi$ and form two equations by equating the real and imaginary parts to zero. This is a much more complicated method and few candidates made any real progress with it. Most candidates using the quadratic formula quoted the correct formula, but some did not substitute into the formula correctly, and many made errors in evaluating the discriminant. Some candidates who reached a correct quotient of complex numbers did not go on to express their answer in the required form.
- (ii) Responses to this part of the question were often poor, and several candidates offered no response. Many candidates were clearly not familiar with complex numbers given in polar form, and some had an axis on their Argand diagrams marked in multiples of $\frac{\pi}{4}$ suggesting that their axes represented the modulus and argument rather than the real and imaginary parts of the number. Those candidates accustomed to working with complex numbers in this form often gave fully correct solutions, although some did not spot any of the simple routes to finding the area of the triangle.

Answer: (i) $-\frac{4}{5} + \frac{3}{5}i$, $-i$ (ii) 10;



Question 10

Most candidates made a correct start in this question by separating the variables and attempting to integrate. A large number of fully correct solutions were seen, but both integrals presented challenges. Some candidates did not appear to recognise the form $\int \frac{f'(y)}{f(y)} dy$ and offered solutions which treated the y^2 as a coefficient, with $\frac{y^2}{y^2} \ln(y^3 + 1)$ seen in their working. More commonly, candidates did not remember that they could use the double angle substitution to find $\int \cos^2 x dx$. For those candidates who did start the integral correctly, many made the error $\int \cos 2x dx = 2 \sin 2x$.

Those candidates who did not realise that they could use the equation given in the question to find the values of x at the turning points often started by differentiating their solution to the differential equation to find an expression for $\frac{dy}{dx}$. Some candidates offered solutions for x in degrees rather than radians, which is incorrect in this context. Candidates with correct integration and correct x values usually completed the question correctly. An error in the coefficient of $\sin 2x$ resulted in solutions which were only fortuitously correct.

Answer: 5.9 48.1

MATHEMATICS

Paper 9709/41

Paper 41

Key Messages

- It is important for candidates to read the question carefully, and answer as required.
- Attention needs to be paid to making sure workings are carried out at a sufficient level of accuracy to ensure the accuracy of the final answer.

General Comments

It is disappointing to report that many candidates did not score some of the marks, simply because the question had not been read carefully. This has happened in the following situations.

- Many candidates assumed from the diagram in **Question 2**, that the forces of magnitudes 30 N and 40 N act in a vertical plane, although the first line of text has a sentence starting with 'Horizontal forces
- Many candidates gave correct statements of $a = -2 \text{ ms}^{-2}$ and $a = -2.5 \text{ ms}^{-2}$ in part (i) of **Question 4**, although the question starts with 'Find the deceleration of each
- Many candidates had a linear combination of the work done against resistance, the change in kinetic energy and the change in potential energy in finding the work done by the driving force in part (i) of **Question 5**. However the question starts with 'A long climb at a constant speed. Thus there can be no change in kinetic energy.
- Many candidates used formulae related to uniformly accelerated motion (as shown in the CIE formulae sheet) in both parts of **Question 7**. However in the stem of part (i) of the question 'P has acceleration $0.6t \text{ ms}^{-2}$ ' and in the stem for part (ii) 'P has acceleration $-0.4t \text{ ms}^{-2}$ ' is stated.

In the first three questions angles are given in terms of the tan, cos and sin respectively. In almost all cases, candidates used these data to find the angles in degrees. These angles in degrees are not needed and almost always introduce approximations instead of exact values. In **Question 2** and **Question 3** the only trigonometric value directly required is the one given in each of the cases. In **Question 1** $\tan \alpha$ is given and $\cos \alpha$ and $\sin \alpha$ are both required. They can be found easily as $15 \div (8^2 + 15^2)^{1/2}$ and $8 \div (8^2 + 15^2)^{1/2}$ respectively.

Comments on Specific Questions

Question 1

Almost all candidates resolved forces vertically and horizontally, but many candidates obtained inaccurate answers because of approximation of $\sin \alpha$ and $\cos \alpha$, and because they used the incorrect trigonometric ratios.

Answer: $F = 1.6$

Question 2

Both parts of this question were attempted well, although some candidates gave the answer for part (ii) as 'the weight is 8 kg.'

Answer: (i) 1000 J (ii) 80 N

Question 3

This question was well attempted, especially part (ii).

Answer: (i) $F = 103$ (ii) 122.5 m

Question 4

- (i) This part of the question was well attempted, although many candidates did not state the decelerations but gave the answers as $a = -2 \text{ ms}^{-2}$ and $a = -2.5 \text{ ms}^{-2}$ for P and Q respectively.
- (ii) Many candidates could not decide a strategy for finding the speeds of P and Q at the required instant.

Answer: (i) 2 ms^{-2} , 2.5 ms^{-2} (ii) 6.09 ms^{-1} , 0.614 ms^{-1}

Question 5

- (i) Candidates who found the driving force and multiplied it by the length of the hill usually found the correct answer for the work done by the driving force. Most candidates who used work/energy calculated the gain in potential energy and the work done against the resistance, then usually added these quantities together to obtain the correct answer.
- (ii) Most candidates obtained a simple equation in d (the required distance) by equating the work done by the engine with the sum of the increase in kinetic energy and the work done against the resistance. Almost all of these candidates found the correct answer for the required distance.

Answer: (i) $4.99 \times 10^6 \text{ J}$ (ii) 1500 m

Question 6

- (i) This part was attempted completely and correctly by almost all candidates.
- (ii) Most candidates obtained the distance travelled by each of the particles while the string is taut. However many candidates did not use $s = ut + \frac{1}{2}gt^2$ with s the distance that A falls from the time the string breaks until A reaches the floor, and $u = -1.6$.

Answer: (i) 4.2 N (ii) 0.6 s

Question 7

- (i) This part was reasonably well attempted by those who recognised the need to use calculus.
- (ii) Very few candidates recognised the need to use $v(10) = 30$ to obtain the constant term of 50 in the expression for $v(t)$ and similarly very few used $s(10) = 100$ to obtain the constant term of $-1000/3$ in the expression for $s(t)$.

Answer: (i) Velocity = 30 ms^{-1} , displacement = 100 m (ii) 194 m

MATHEMATICS

Paper 9709/42

Paper 42

Key Messages

- Candidates need to be careful to read the question in detail and answer as indicated.
- A considerable proportion of the total marks in this paper depend on giving answers which are correct to 3 significant figures. In order to gain as many marks as possible all working should be either exact or correct to at least 4 significant figures.

General Comments

The paper was generally well done by many candidates. The presentation of the work was good in most cases. Surprisingly, overall the candidates made better attempts at the questions that occurred later in the paper than the ones which occurred towards the beginning of the paper.

Candidates should be advised not to divide their written answer page vertically as this often makes it much more difficult to follow the candidate's work and their arguments.

Some candidates lost marks due to not giving answers to 3 sf as requested and also due to prematurely approximating within their calculations leading to the final answer, particularly in **Question 4**. Candidates should be reminded that if an answer is required to 3 sf then their working should be performed to at least 4 sf.

In **Questions 1, 3 and 4**, sines and tangents of angles were given in the question. This should indicate to candidates that there is no necessity to use a calculator in order to determine the angle itself. However, many candidates often then proceeded to find the angle and immediately lost accuracy and marks.

One of the rubrics on this paper is to take $g = 10$ and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve the correct answer unless this value is used.

Comments on Specific Questions

Question 1

Most candidates attempted to resolve along the direction of the inclined plane and hence produced an equation which did not involve the normal reaction. This was the best approach but some candidates mixed up sine and cosine when resolving. Often the angle itself was found during the calculations rather than using the given values of $\sin \alpha$ and this led to inaccuracies in the later calculations.

Answer: $T = 2.5 \text{ N}$

Question 2

Many candidates produced good answers to this question using the principle of Work/Energy. An error which some candidates made was to use incorrect signs when applying this method. A significant number of candidates assumed incorrectly that this was a constant acceleration problem. The resistive force was not given as constant but only in terms of work done against resistance over the whole period of the motion. Candidates should be aware of such wording.

Answer: $\alpha = 19.4$

Question 3

- (i) This part of the question was well done with most candidates equating the driving force of 25 N to the resistive force $4k$ to produce the correct result.
- (ii) In this part many continued incorrectly by using 25 N as either the driving force or the resistive force rather than using $100/v$ and kv respectively as was given in the question and consequently marks were lost at this stage.

Answer: $v = 2.08 \text{ ms}^{-1}$

Question 4

Most candidates followed the guidance given in the question by resolving forces parallel to and perpendicular to the plane. In questions such as this it is vital to begin by setting up a diagram showing all the forces acting on the system which in this case were P , the weight (mg), Friction (F) and the normal reaction (R). By resolving as suggested each of F and R can be expressed in terms of components of P and mg . However, some candidates equated R merely to a component of the weight. Again some accuracy was lost in many cases where the angle was found rather than using $\tan \alpha$ as given.

Answer: $P = 6.12$

Question 5

- (i) This part of the question was generally well done with candidates integrating the given expression for velocity and using the correct limits. A few candidates assumed that the v - t diagram was in fact a trapezium and for this part merely found the area of a triangle in spite of the fact that v was given as a quadratic expression.

Answer: Distance = 8.33 m

- (ii) The distance from $t = 5$ to $t = 45$ is determined simply as the area of a rectangle. Many found this straightforward but a number of candidates either did not attempt this section of the motion or attempted it using integration of one of the quadratic expressions. The final section was generally well started by integrating the expression but many lost marks for incorrect calculations when using the limits or in some cases using incorrect limits. The majority of candidates knew how to evaluate the average speed for the journey.

Answer: Total distance from O to A = 117 m, Average speed = 2.33 ms^{-1}

Question 6

- (i) Most candidates made good attempts at this part of the question with the majority writing down Newton's second law for each particle and finding the tension T from elimination and then multiplying by 1.2 to determine the work done. A few candidates used the Work/Energy principle to achieve the result. Overall this part was well done.

Answer: Work done by tension = 7.68 J

- (ii) Again most candidates made reasonable attempts at this part of the question. The usual method was to determine the velocity of A when B reached the floor using the value of $a = 6$ from their equations in part (i). After this stage of the motion A moves under gravity. The most common error was that candidates either used $a = 6$ for both parts of the motion or used $a = g$ for both parts. Having found the distance h moved by A under gravity a large number of candidates then stated that the height of A above the floor was $1.2 + h$ rather than $2.4 + h$.

Answer: Greatest height above floor = 3.12 m

Question 7

- (i) This part was well done by almost all candidates. An error which was occasionally seen was a missing factor of $1/2$ in the evaluation of the area of one of the triangles.

Answer: 9.4 m

- (ii) Again this part was generally well done. Candidates should remember that an acceleration of -0.1 is the same as a deceleration of 0.1 and hence should remember to take extra care with the signs.

Answer: 0.08 ms^{-2} and 0.1 ms^{-2}

- (iii) A good number of candidates correctly answered this part by applying Newton's second law to the elevator and box system. There were some misunderstandings with sign errors.

Answer: Stage 1: $T = 9072 \text{ N}$, Stage 2: $T = 9000 \text{ N}$, Stage 3 $T = 8910 \text{ N}$

- (iv) Very few candidates correctly answered this part. Some considered the box but assumed that it was acted upon by the tension rather than merely by its weight and the required reaction between box and elevator. It was expected that candidates would attempt to apply Newton's second law to the box in the form $R - 100g = 100a$ where R is the required reaction between the elevator and the box.

Answer: Greatest reaction force = 1008 N , Least reaction force = 990 N

MATHEMATICS

Paper 9709/43

Paper 43

Key messages

- When a question involves the sine or cosine of an angle, candidates should consider either using exact sine / cosine values if possible or beware when using angle approximations which may lead to a loss of accuracy (e.g. **Question 1**, **Question 3** and **Question 4**).
- Candidates should check that all forces have been considered when resolving forces in equilibrium or when forming an equation either using Newton's Second Law or using work and energy (e.g. **Question 2**, **Question 4** and **Question 6**).

General comments

The paper allowed candidates to show what they knew and most candidates attempted all questions. Many very good scripts were seen and the majority of candidates were able to answer at least a part of each question. On this paper **Question 5** was the least well answered.

Comments on specific questions

Question 1

- (i) The answer -6 was given in this question. Candidates usually resolved along the plane, but the negative in the given answer was not always shown or fully justified e.g. $0.32g + 0.28g = 6$, $a = -6 \text{ ms}^{-2}$. The use of decimal or fraction values for $\sin\alpha$ and $\cos\alpha$ led to the exact value -6 whereas the use of the approximations $\sin 16.3^\circ$ and $\cos 16.3^\circ$ led to 6.01 , rather than the required answer.
- (ii) Most candidates gained full marks using either one or two acceleration formulae to obtain the required distance, even if part (i) was not attempted. A significant number of candidates gave the longer solution finding the time taken first.

Answer: 2.43 m

Question 2

This question was often well answered. The best solutions used Newton's Second Law to find the acceleration of the particles and then used $v^2 = 2as$ as to obtain the speed. Some candidates assumed that the acceleration was $g \text{ ms}^{-2}$ despite the tension in the string. Others who used an energy equation to find the speed of B also omitted to consider the work done due to the tension and erroneously found the speed to be 5.66 ms^{-1} .

Finding the value of h presented more of a problem. Some candidates used the acceleration found for the connected particles (5 ms^{-2}) instead of $a = g \text{ ms}^{-2}$. Others formed a suitable equation to find ' $s = 0.8$ ' but were not always sure of the connection between s and h concluding e.g. $h = 0.8$. Those who used $a = 5$ for the 'slack' motion found ' $s = 1.6$ ' and concluded e.g. $h = 3 - 1.6 = 1.4$ (omitting the first stage of motion) or else calculated $3 - 1.6 - 1.6 = -0.2$ and stated $h = 0.2$.

Answer: 4 ms^{-1} 0.6

Question 3

Many candidates gained full marks, usually by resolving the forces vertically and horizontally and then solving the resulting simultaneous equations. Those who worked with fraction values for \sin/\cos PAB and \sin/\cos ABP often produced neat and concise solutions whilst those who worked with decimal approximations sometimes lost accuracy. Some candidates assumed that the tension in the two strings was equal, some used 1.05 kg instead of 10.5 N and a few believed that the angle at B was a right angle. A labelled diagram would have made some solutions easier to follow as it was not always clear which angles were being used in the equations formed. Some candidates worked with each string separately attempting e.g. $T_{AP} = 10.5/\sin BAP$ and thus gained no marks.

Answer: 6.5 N 10 N

Question 4

- (i) This part of the question was often well answered. Loss of marks was regularly a result of early approximation leading to a loss of accuracy in the value for μ e.g. $10/298 = 0.0336$. Some candidates misunderstood the direction of the 40 N force, inclining it at an angle α to the plane. Some omitted the component of weight to get $\mu = 40/R = 0.134$ whilst others had a directional error leading to the frictional force as $40 + 30$ instead of $40 - 30$.
- (ii) Candidates were expected to compare the weight component (30 N) acting down the plane and the frictional force (10 N) acting up the plane. A clear explanation for non-equilibrium was frequently given. However, incorrect or incomplete explanations were also frequent and sometimes based on the false assumption that the frictional force was still acting down the plane.

Answer: 0.0335 Not in equilibrium

Question 5

This was the least well answered question. The main difficulty resulted from the misinterpretation of T_2 and T_3 as the specific times at which the acceleration ended (T_2) and at which the deceleration began (T_3) rather than the times taken for the second and third stages of motion.

- (i) Since the magnitude of the deceleration was greater than that of the acceleration it was expected that the third line segment would be steeper than the first. Often the sketch together with a segment of the t axis appeared as an isosceles trapezium. It was not unusual for candidates to omit to find T_1 and T_3 in terms of V as required, whilst those who misinterpreted T_2 and T_3 frequently found $T_3 = -V$ and seemed unsurprised by the negative value for the time taken.
- (ii) Although candidates understood that the distance travelled was equivalent to the area under the graph, many were unable to achieve the quadratic equation given either following misinterpretation of T_2 and T_3 or through difficulty with algebraic manipulation. The use of one trapezium rather than two triangles and one rectangle often gave a more straightforward solution. Those who did not form the quadratic equation were still able to attempt solution for V but sometimes gave two solutions instead of selecting the appropriate one.

Answer: $T_1 = V \div 0.3$ $T_3 = V$ 24

Question 6

- (i) This solution required the application of both Newton's Second Law and driving force = P/v . In some cases only one of acceleration and velocity was found even though both solutions depended on $P/v - R = ma$ or equivalent. An acceleration of 0.72 ms^{-2} was a common incorrect answer obtained when the resistance was not considered.
- (ii) Fully correct solutions were often seen, more usually from a consideration of work and energy than from resolving forces along the plane and applying $F = ma$. When errors occurred with the work/energy method it was mainly due to a missing term, often the work done against resistance, or from including a force rather than the work done by the force e.g 5800 rather than 5800×500 . When using the Newton's Law method, the component of weight down the plane was sometimes missing ($5800 - 4800 = 12500a$) leading to an acceleration of 0.08 ms^{-2} rather than a deceleration of 0.32 ms^{-2} .

Answer: 0.336 ms^{-2} 20 m

Question 7

Most candidates recognised that this question needed integration and it was a small minority that attempted to use either differentiation or constant acceleration formulae.

- (i) This part of the question was generally well attempted with integration used to find the distance travelled and hence the value of k_1 . 0.1 was sometimes seen as the solution to the equation $1800k_1 - 360 = 540$ which meant that a correct value for k_2 could not then be found. Candidates who equated $v_1(60)$ to $v_2(60)$ were usually successful in finding k_2 whilst those who integrated v_2 usually ignored any constant of integration and found $k_2 \neq 12\sqrt{60}$.
- (ii) Candidates experienced some difficulty working with two stages of motion, sometimes misinterpreting which distance was required. It was common to see $s = 24\sqrt{(60t)}$ assuming $t = 0$, $s = 0$ rather than $t = 60$, $s = 540$. Those who assumed $t = 0$, $s = 540$ erroneously found $s = 24\sqrt{(60t)} + 540$. The rearrangement of k_2/\sqrt{t} to $k_2t^{1/2}$ rather than $k_2t^{-1/2}$ before integration was another cause of error.
- (iii) Candidates were expected to equate their part (ii) answer to the total distance travelled (1260 m) in order to find the time taken. A common error was to subtract 540 from 1260 and equate their part (ii) answer to 720 considering only the distance travelled after 60 seconds.

Answer: 0.5 $24\sqrt{(60t)} - 900$ 8 ms^{-1}

MATHEMATICS

Paper 9709/51

Paper 51

Key Messages

- Candidates need make sure they do not make approximations during their working, as this can impact on the accuracy of the final result.
- Candidates need to be careful to read the question in detail and answer as indicated.

General Comments

Generally the work seen was neat and well presented. Some candidates are still not obeying the instruction to give answers to 3 significant figures. The easier questions proved to be **1**, **3** and **4(i)**, while the harder questions proved to be **2(i)**, **4(ii)**, **5(ii)** and **6(i)**.

Again it is necessary to remind candidates to use the formula booklet provided.

The use of $g = 9.8/9.81$ was very rarely seen.

Comments on specific questions

Question 1

Most candidates scored the 2 marks for this question. The principles of motion in a circle were clearly understood.

Answer: $T = 12.5 \text{ N}$

Question 2

- (i) Most candidates managed to find the centre of mass of the arc. A minority of the candidates used the incorrect formula. Too many candidates thought that this was the required answer. A moment equation was then needed for the whole frame.
- (ii) Most candidates attempted to use $\tan \alpha = (\text{centre of mass of the frame})/0.6$, where α is the required angle between AC and the vertical.

Answers: (i) 0.233 m (ii) 21.3°

Question 3

- (i) This part of the question was well done.
- (ii) Most candidates attempted to integrate and then went on to solve the question. Some candidates only had $v =$ and not $v^2 =$ after their integration. A number of candidates thought that e^0 was zero rather than 1.

Answers: (i) $vdv/dx = 5e^{-x} - 3x^2$ (ii) $v = 5.35 \text{ ms}^{-1}$

Question 4

It was pleasing to see how the candidates tackled this projectile question.

- (i) The speed was often found correctly by using Pythagoras's Theorem with the vertical and horizontal components of the velocities. The 40° angle for the direction was often seen but with no explanation to say that it was below the horizontal.
- (ii) Too many candidates gave their equation as $2 = (14 \sin 60^\circ)t - gt^2/2$ instead of $-2 = (14 \sin 60^\circ)t - gt^2/2$.

Answers: (i) Speed = 9.14 ms^{-1} , Direction 40° below the horizontal. (ii) Time = 2.58 s

Question 5

- (i) This part of the question was generally well done. Most candidates resolved vertically to find the tension, and then used Newton's Second Law horizontally to find the speed of P. Sometimes the radius of the circle was incorrectly found.
- (ii) Too many candidates used the tension found in part (i). Other candidates did not realise that the tension in BP was zero.

Answers: (i) Speed = 4.10 ms^{-1} (ii) Angular speed = 5.59 rad s^{-1}

Question 6

- (i) The trigonometry needed to find the distances required to set up the moment equation often proved to be very difficult.
- (ii) This part of the question was successfully done by many candidates. Some candidates made a sign error when trying to calculate the normal reaction. $R = 260 - 146 \sin 30^\circ$ was often seen instead of $R = 260 + 146 \sin 30^\circ$.

Answers: (i) Tension = 146 N (ii) Coefficient of friction = 0.380

Question 7

Candidates appeared to be well prepared for an elastic string problem.

- (i) Many candidates scored maximum marks for this part of the question. The correct energy equation was often seen. A few candidates failed to solve the resulting quadratic equation.
- (ii) Again this part of the question was well done.
- (iii) Too many candidates used 96% of the Kinetic Energy instead of 4% in the final equation.

Answers: (i) 1.31 m (ii) 4 ms^{-1} (iii) OP = 0.768 m

MATHEMATICS

Paper 9709/52

Paper 52

Key Messages

- Candidates need make sure they do not make approximations during their working, as this can impact on the accuracy of the final result.
- Candidates need to be careful to read the question in detail and answer as indicated.

General Comments

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Question 6

- (i) The trigonometry needed to find the distances required to set up the moment equation often proved to be very difficult.
- (ii) This part of the question was successfully done by many candidates. Some candidates made a sign error when trying to calculate the normal reaction. $R = 260 - 146 \sin 30^\circ$ was often seen instead of $R = 260 + 146 \sin 30^\circ$.

Answers: (i) Tension = 146 N (ii) Coefficient of friction = 0.380

Question 7

Candidates appeared to be well prepared for an elastic string problem.

- (i) Many candidates scored maximum marks for this part of the question. The correct energy equation was often seen. A few candidates failed to solve the resulting quadratic equation.
- (ii) Again this part of the question was well done.
- (iii) Too many candidates used 96% of the Kinetic Energy instead of 4% in the final equation.

Answers: (i) 1.31 m (ii) 4 ms^{-1} (iii) OP = 0.768 m

MATHEMATICS

Paper 9709/53

Paper 53

Key Messages

- Candidates need to make full use of the supplied list of formulae.
- Marks depend on giving answers which are correct to 3 significant figures. In order to gain as many marks as possible all working should be either exact or correct to at least 4 significant figures.

General Comments

Questions 6 and 7 were found to be most challenging by the candidates. The presentation of the work was good in most cases.

The drawing of good, clear diagrams can often help candidates.

The use of $g = 9.8/9.81$ was very rarely seen. $g = 10$ should be used as indicated on the front page of the question paper.

Comments on Specific Questions

Question 1

This question was usually well done and many candidates scored all 3 marks.

Answer: Speed = 6 ms^{-1}

Question 2

- (i) Many candidates scored both marks for this part of the question.
- (ii) The use of $a = 0$ was used to find x when the velocity of P was greatest.
- (iii) Most candidates attempted to integrate and then used their value of x found in the previous part. Unfortunately after integrating some candidates only had $v = \dots$ instead of $\frac{v^2}{2} = \dots$

Answers: (i) $v \text{d}v/\text{d}x = 10 - 0.03x^2$ (ii) $x = 18.3$ (iii) Greatest value of $v = 15.6$

Question 3

Many candidates resolved vertically and also used Newton's Second Law horizontally to set up 2 equations in R and θ , where R was the required force and θ was the semi-vertical angle.

Answer: Semi-vertical angle = 45° , magnitude of the force exerted by the surface of the cone = 7.07 N

Question 4

- (i) Newton's Second Law was often used to find the acceleration as a function of t . After correctly integrating to find v as a function of t some candidates found an incorrect value for c .
- (ii) The equation $t^2 - 50t + 15 = 0$ was solved to give 2 values of t . It was necessary to say that the correct time was 0.302 s. Too many candidates either left both values of t without specifying the correct one or they quoted the larger value.
- (iii) Most candidates simply quoted the other value of t from the previous part. The correct value could be calculated by equating the tractive force and the friction.

Answers: (i) $a = 0.12t - 3$, $v = 0.06(t^2 - 50t + 15)$ (ii) $t = 0.302$ (iii) $t = 25$

Question 5

- (i) This part of the question was generally well done by most candidates.
- (ii) $20 = (15\sin 30^\circ)t - gt^2/2$ was too often seen instead of $-20 = 15(\sin 30^\circ)t - gt^2/2$

Answers: (i) Speed = 25 ms^{-1} ; direction is 58.7° to the horizontal (ii) OP = 42.5 m

Question 6

- (i) Very few candidates were able to set up the 5 term energy equation. Initially it was necessary to find the extension and then to state that the particle moves down a certain distance. After doing this the energy equation could be set up and solved to find the greatest speed.
- (ii) It was necessary to show that insufficient energy was available for the particle to reach O. This is done by saying that PE gain = $0.4g \times 0.82 = 3.28 \text{ J}$ and at the initial position $\text{KE} + \text{EE} = 0.4 \times 1.5^2/2 + 50(0.82 - 0.8)^2/(2 \times 0.8) = 0.4625 \text{ J}$. Hence there is not enough energy.

Answers: (i) Greatest speed of P is 1.60 ms^{-1} (ii) Insufficient energy

Question 7

- (i) Some candidates used an incorrect formula to find the centre of mass of the cone. A formula booklet is supplied and should be used. A moment equation should be set up in order to solve this part of the question.
- (ii) This part of the question was usually well done with candidates scoring both available marks.
- (iii) $F = \mu R$ was used correctly by most candidates to find the coefficient of friction, μ .
- (iv) This part of the question proved extremely difficult. Candidates were unable to argue for or against toppling or sliding.

Answers: (i) Centre of mass of the solid above the surface = 0.275 m (ii) $F = 37.5$ (iii) Coefficient of friction = 0.625 (iv) Solid does not slide but topples

MATHEMATICS

Paper 9709/61

Paper 61

Key Messages

To do well in this paper candidates must work with 4 significant figures or more in order to achieve the accuracy required. Candidates should also show all working, so that in the event of a mistake being made, credit can be given for method; a wrong answer with no working shown scores no marks. Candidates should label graphs and axes including units, and choose sensible scales.

General comments

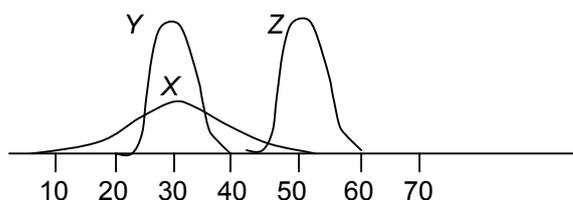
Many of the candidates who took this paper did not appear to have covered the full syllabus. Most candidates appeared to find this paper quite challenging.

Comments on specific questions

Question 1

This question asked candidates to draw three normal distributions, given the mean and standard deviation in the form $X \sim N(30, 49)$. Approximately half the candidates who attempted this question drew normal curves but failed to appreciate that the standard deviation was 7 and since at least 99% of a normal curve is within 3 standard deviations from the mean, the graph should range from approximately 10 to 50 on the horizontal axis. The second graph $Y \sim N(30, 16)$ only had a standard deviation of 4 so should be much thinner and since the area under the curve is 1, should rise much higher.

Answer:



Question 2

This was a straightforward conditional probability question with the information set out in a table. Those candidates who recognised this as such, drew a tree diagram and used the conditional probability formula gained full marks. A very large percentage did not use the formula for conditional probability questions and so were unable to gain more than 1 mark maximum in this question.

Answer: 88/145 (0.607)

Question 3

Candidates who knew that the variance formula is the same for coded data as uncoded data found this question straightforward. Approximately half of those who attempted this question found $\Sigma(x - 5)$ successfully, but only a few managed to equate variances for coded and uncoded data and solve.

Answers: 26, 257

Question 4

This question asked for a box-and-whisker plot. Candidates mostly found the median and quartiles successfully but were unable to draw a box-and-whisker plot with straight lines, a scale and labels (house prices, \$000's) thus losing 2 of the 4 marks. Parts (ii) and (iii) were usually done well.

Answers: (ii) \$957 000 and \$986 000.

Question 5

This question was well done by those candidates who had covered the normal distribution in their lessons. They standardised correctly, looked up backwards in the table, and solved for c. Part (ii) involved finding a probability from the normal distribution and then applying the binomial to this probability. Again, this was well done with many candidates (of those who attempted this question) gaining full marks.

Answers: 9.14, 0.159

Question 6

It was pleasing to see that many candidates recognised that there were 3 options for this part, each of which was a product of combinations. Many gained full marks and of those who did not, most managed to get started and pick up a few method marks. Part (ii) needed a little thought but again, many candidates managed to answer this correctly, similarly with part (iii).

Answers: 13580, 288, 240

Question 7

The subject matter in this question was straightforward, and almost everybody managed to achieve good marks on parts (i), (ii) and (iii). Part (iv) identified the candidates who could not remember the difference between independent and exclusive events but a large number gained full marks for this question overall.

Answer: (iii) $5/32$, $7/32$, $3/32$

MATHEMATICS

Paper 9709/62

Paper 62

Key Messages

To do well in this paper, candidates must work with 4 significant figures or more in order to achieve the accuracy required. Candidates should also show all working, so that in the event of a mistake being made, credit can be given for method; a wrong answer with no working shown scores no marks.

General comments

This paper enabled many candidates to demonstrate their sound knowledge of the syllabus. There were also many candidates however who appeared to recognise the words like normal distribution, binomial distribution, arrangements, mean and standard deviation, but had not studied them in any depth and so were unable to complete the questions successfully. Some calculator work was inaccurate; many candidates lost up to 5 marks because they wrote the correct unsimplified expressions but then appeared to press the wrong buttons on their calculators and thus lost the final accuracy mark.

Comments on specific questions

Question 1

It was pleasing to see the number of candidates who completed this question correctly. It was good that many candidates could cope with the minus sign. Some ignored it and gained a method mark for standardising but nothing more. There were many instances of careless reading of the tables though, with $\Phi(1.219)$ being read as $\Phi(1.0219)$, thus losing the final accuracy mark. Another common slip, for those who had successfully found $\Phi(1.219)$ to be 0.8886, was to subtract it from 1 writing 0.114 instead of 0.1114, thus losing the final accuracy mark.

Answer: 0.111

Question 2

- (i) With a few exceptions candidates made effective use of a tree diagram and chose the correct combination of probabilities: $0.35 \times (1 - 0.8)$
- (ii) A tree diagram was helpful in avoiding a common error in calculating the probability of Mohit spending less than \$50. This error was to omit the probability he stayed at home and led to using $0.25 \times 0.3 + 0.35 \times 0.2$ instead of $0.25 \times 0.3 + 0.35 \times 0.2 + 0.4 \times 1$ for the denominator of the conditional probability.

It was pleasing that many candidates recognised that there had to be a fraction and managed to evaluate some of the probabilities correctly.

Answers: 0.07, 0.128

Question 3

- (i) Standardising the amount of fibre proved an easy task. Using the normal tables in reverse was less so. A diagram should have made it clear that the z-value had to be equal to the negative value of $\Phi^{-1}(0.81)$. This value is 0.878 but a significant number of candidates carelessly wrote down 0.842 which is $\Phi^{-1}(0.80)$.
- (ii) At least one (out of twelve) was correctly calculated by many as $1 - P(0)$, although some chose to calculate $1 - P(0) - P(1)$, not fully comprehending that the phrase “at least 1” means ‘one or more’ i.e. 1,2,3,4,....., not ‘more than one’ i.e. 2,3,4,5,..... Those who calculated $1 - p(1)$ forgot that this includes the probability of zero packets and therefore gained no marks. In part (ii) the answer was $1 - (0.81)^{12}$ which should not have been too difficult to evaluate with the aid of a calculator. However, too many candidates read $(0.81)^{12}$ to be 0.078 or 0.079 instead of 0.0798 and thus lost the accuracy mark at the end.

Answers: 34.2, 0.920

Question 4

It was pleasing to see that almost everybody managed to obtain the correct given frequency. Those who knew how to find the mean of a grouped frequency table went on and correctly evaluated the rest of the frequencies before substituting in the appropriate formula for the mean. There were many interpretations of ‘x’ in the formula $\bar{x} = \frac{\sum fx}{\sum f}$, including class widths, upper class boundaries, lower class boundaries, semi-class-widths, as well as the correct mid-interval. This meant that many candidates only gained 3 marks for this question. The last part (iii) was well answered by many candidates. It was expected that candidates would type their numbers in to their calculators and use SD mode to calculate the mean and variance, writing down the numbers for $\sum fx$ from their calculators. Thus this question should have been a quick question, whereas in fact it took many candidates a lot of unnecessary time. In (iii) recognising that using the mid point of each class **assumes** that all the data in that class have that value is what needed to be explained.

Answers: 213, 46.5

Question 5

This question frequently gained full marks showing that the binomial distribution and the normal approximation to the binomial were well understood. There were still those candidates who used a normal approximation inappropriately in part (i) and gained no credit, or tried to calculate and sum the binomial probabilities in part (ii) and made little progress with the amount of computation which had to be done. In part (i) the question asked for “between 4 and 6 inclusive”, which is an alternative way of saying 4, 5 or 6 but many candidates could not see this and chose to evaluate $P(X < 6 \text{ or } 7) - P(X < 4 \text{ or } 5)$ or worse. Even those who evaluated the correct 3 probabilities usually lost the final mark because they worked with 3 figures and thus lost the final accuracy. Others wrote the answer 0.076 which is only 2 significant figures, and so lost the final accuracy mark.

Answers: 0.0763, 0.892

Question 6

This question produced many good answers with most candidates scoring full marks for (i) and (ii). Some candidates thought that “not having all 4 vowels together” was the same as having each vowel separately, not taking into account that it was also possible to have 2 vowels together and 2 separate and so on. This approach gained 1 mark out of the 3, as they showed some knowledge. The neatest method was to evaluate the number of arrangements with all 4 vowels together and subtract from their answer to part (i).

Answers: 1663200, 30240, 1622880, 10

Question 7

This last question was very easy once candidates had understood the situation, and a large majority gained full marks in parts (i) and (ii). As usual, there was confusion over what was required for the justification of independence and exclusivity. Numbers (probabilities) needed to be calculated and tested against the definitions of independence and exclusivity. Wordy waffle was not acceptable. An alternative for part (iv) was a sentence saying 'there is an overlap between R and S because card 3 has to be in both', which again needed to have the example of number 3 to gain the mark.

Answers: (ii) $P(9) = 1/30$, $P(10) = 3/10$, $P(11) = 1/2$, $P(12) = 1/6$.

MATHEMATICS

Paper 9709/63

Paper 63

Key Messages

Candidates should be encouraged to show all necessary working. A significant number of candidates did not show sufficient working to make their approach clear.

To do well in this paper, candidates must work to 4 significant figures or more to achieve the accuracy required.

Candidates should be encouraged to sketch normal distribution graphs where appropriate.

General Comments

Answers to **Questions 1, 6 and 7** were generally stronger than answers to others questions.

The majority of candidates used the answer booklets provided effectively, however a number did not utilise the available space appropriately by answering the entire paper on a single page.

A number of candidates made more than a single attempt at a question and then did not indicate which their submitted solution was. A small number of candidates did not replace deleted solutions.

Comments on Specific Questions

Question 1

Most candidates answered this question well. Many candidates identified that there were errors on both axes and used accurately the terms 'frequency density' and 'continuous data' as part of their explanation. A large number of candidates also provided examples to clarify the necessity to use class boundaries on the horizontal axis. A significant number of candidates restated the horizontal axis error. A few candidates attempted to give more than two reasons as to why the histogram was not correct.

Question 2

Most candidates used correctly the normal distribution formula. A very small number of candidates incorrectly used a continuity correction. Good solutions often had a simple sketch of the normal distribution with the mean and required sizes of flowerpots marked, enabling candidates to correctly evaluate the appropriate tail probabilities required for the probability of choosing a single flowerpot between 13.6 cm and 14.8 cm. Most candidates then attempted the Binomial distribution required to answer the question, using their calculated probability correctly. Candidates were expected to use a value to four significant figures in this calculation.

Answer: 0.252

Question 3

Most candidates identified that the $p(\text{medium or large}) = p(\text{not small}) = 0.85$.

- (i) Good solutions included the correct application of the Binomial distribution with the required values. A number of candidates interpreted the question to include 12 mangoes in the required range when calculating the tail. Very few candidates summed the 12 correct binomial expressions,

the majority using the anticipated approach of $1 - p(12, 13 \text{ or } 14 \text{ mangoes})$. Some candidates used the normal distribution here which could gain no credit.

- (ii) Most candidates produced an appropriate inequality. Good solutions had clearly expressed use of logarithms and algebraic manipulations. A few candidates attempted to use Trial and Improvement. There were a significant number of solutions seen with inconsistent use of inequalities and the resulting incorrect interpretation of the candidate's final answer.

Answers: (i) 0.352 (ii) 14

Question 4

Many candidates gained full credit for this question. Good solutions used the weighted mean in (i) and applied the variance formula correctly in (ii).

- (i) Almost all candidates attempted this question. The most common error was to calculate the simple mean of the two values, although a few candidates simply added the 2 means and then divided by 45.
- (ii) Most candidates recognised that the variance needed to be calculated, but a few evaluated the standard deviation formula throughout. The most common initial error was to evaluate $(\sum x_n)^2$. A number of candidates used their rounded answer for (i) which would lead to an inappropriate value.

Answers: (i) 163 g (ii) 3100 – 3120 g

Question 5

Candidates found this a most challenging question. Good solutions had clear workings including the algebraic manipulations required. Most candidates attempted to use the Normal Distribution. However, many candidates could benefit from additional focus on the correct interpretation of the Normal Distribution Function table.

- (a) Most candidates were able to standardise correctly, although a number were unable to resolve the required range. Although most candidates used the tables correctly, many did not find the correct z value as the question was not interpreted appropriately.
- (b) The good solutions seen often resolved the standardisation prior to any attempt at substitution. Many candidates attempted to substitute and standardise simultaneously, with some complex algebra resolution which was then problematical to solve. Many candidates were inconsistent with their interpretation of their calculation, and many solutions had sign errors which affected their final answer.

Answers: (a) 4.31 (b) $\sigma = 2.90, \mu = 3.36$

Question 6

Most candidates had a clear understanding of the requirements of parts (i), (ii) and (iii). However many attempted complex methods to part (iv) without ensuring that their outcomes were in proportion to their previous answers.

- (i) Most candidates recognised that effectively there were 8 letters to rearrange, as the R was fixed. Good solutions recognised the impact of the repeated letters and divided by $3!2!2!$.
- (ii) Good solutions interpreted the information so that there were only 5 objects being rearranged. However, a number of candidates correctly considered the arrangement of the individual letters in each block and although more complex calculations were seen, the correct answer was obtained.
- (iii) Many candidates identified that there were $5!$ combinations for the consonants, however a number of candidates assumed that there were ${}_6P_4$ rather than $4!$ arrangements for the vowels. The repetition of the letters was generally handled correctly. A small number of candidates incorrectly summed their two outcomes, rather than multiplied.

- (iv) Nearly all candidates attempted to evaluate the selection of 2 Gs and 3 Gs separately, with most summing to combine their answers. However, the best solutions were where candidates identified that there were very limited possible outcomes and simply listed these. A small number of candidates calculated that with 3 Gs there were ${}_4C_1$ possible outcomes but were subsequently unable to manage the impact of the repeated letters for 2 Gs.

Answers: (i) 1680 (ii) 120 (iii) 120 (iv) 12

Question 7

This question was attempted successfully by almost all candidates. Some candidates penalised themselves by not applying the instruction within the question that the digits were chosen without replacement. Good candidates constructed a full tree diagram before attempting part (i) and then used it throughout.

- (i) Almost all candidates identified and summed the probabilities of the 3 correct pairs of digits, although a few candidates were inconsistent in their evaluation of $P(1,1)$, using $1/9 \times 1/8$. A small number of candidates successfully used outcome space diagrams, although these were often with replacement identified.
- (ii) A large number of candidates correctly determined that there were 2 different orders of achieving the required outcome. Good candidates used their tree diagram to identify all the appropriate outcome pairs, others summed their $P(5 \text{ and not } 5)$ with $P(\text{not } 5 \text{ and } 5)$. A significant minority only considered the first option.
- (iii) Good candidates stated and applied correctly the conditional probability formula. There was some inconsistency in the calculation of $P(5 \cap \bar{5})$, with the candidates answer to part (ii) being frequently used. A number of candidates did not consider all possible combinations required to fulfil $P(\bar{5})$.
- (iv) Most candidates submitted a table of probabilities containing only probability attempts for zero, one or two 5s. A small number of candidates also included a value for three 5's which was outside the remit of the question. Good candidates recognised that parts (i) and (ii) provided the answers for one and two 5's and used the sum of probabilities being 1 to calculate the final value. The majority of candidates attempted to calculate again each value, which was not always consistent with their previous work. Most candidates did apply the sum of the probabilities being one to their solution.

Answers: (i) $5/18$ or 0.278 (ii) $1/2$ or 0.5 (iii) $3/8$ or 0.375 (iv) $P(0) = 5/12$ or 0.417, $P(1) = 1/2$ or 0.5, $P(2) = 1/12$ or 0.0833

MATHEMATICS

Paper 9709/71

Paper 71

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. In general, candidates scored well on **Question 3(i)**, **Question 4(i)**, **Question 5(i)** and **(iii)** and **Question 7**, whilst **Question 6** proved particularly demanding.

Timing did not appear to be a problem for most candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many good and complete answers.

Comments on specific questions

Question 1

Many candidates realised that the appropriate approximating distribution was Po ($1/30$). However, in many cases marks were lost due to premature rounding. Candidates used 0.03 or 0.033 rather than keeping to $1/30$ or 0.0333. It is important that candidates are aware of the accuracy level required. If $1/30$ was not stated and only 0.3 seen, this would have caused the loss of 2 marks.

Some candidates did not use an approximating distribution, but calculated the probability using the Binomial Distribution; others incorrectly attempted a Normal approximation.

Answer: 0.0328

Question 2

Most candidates were familiar with the concept of a confidence interval and used a correct z-value in combination with the standard deviation and sample size. However, a quite significant proportion did not immediately recognise the fact that the width of the interval was not dependent on a sample mean, and were unable to manipulate their confidence interval into an equation or inequality concerning the width. Candidates that were able to set up the required equation (or inequality) often solved it correctly to find $n = 19.2$. Having reached the value of 19.2, it was then necessary to realise that the smallest sample size required was 20. The required smallest sample size was often given as 19 rather than 20.

Answer: Smallest n is 20

Question 3

Many candidates successfully found unbiased estimates for the population mean and variance. There were only a few cases where the candidate found the biased variance thinking that this was the answer required. Some candidates confused the two alternative formulae for the unbiased estimate of the population variance, but this was not as prevalent as has been seen in the past.

Part (ii) was also better attempted than questions of a similar nature on previous examination papers, though weaker candidates still struggled to include all the necessary steps. Candidates generally remembered to state their Null and Alternative Hypotheses, and most used a correct two tail test. It was pleasing to see many candidates clearly showing the required comparison (either as an inequality statement or on a clearly labelled diagram) and in most cases a valid comparison was made. A suitable conclusion with no contradictory statements, and preferably in the context of the question, was then required.

Answer: 2866 4126.53

No evidence that the mean distance changed.

Question 4

Part (i) of this question was well attempted. Many candidates used the correct parameter of 2.8 and calculated $P(0) + P(1) + P(2)$, successfully reaching the required answer. However, there were some candidates who did not adjust λ for 150 minutes to λ for 10 hrs (600 minutes), and some misinterpreted 'at most 2' causing the omission of one of the required terms or an incorrect calculation of $1 - (P(0) + P(1) + P(2))$.

In part (ii) many candidates were able to apply part of the method required, but only the most able candidates reached a fully correct final answer. Many candidates realised that the use of natural logarithms was required to solve their inequality, so method marks could be gained here. However, the full calculation required in order to give the requested time period, in minutes, was not always carried out. Many candidates struggled with the complexity of the parameter to be used in the Poisson distribution. There were also many instances of inaccuracy caused by premature approximation of $\ln 0.99$.

Answer: 0-469

Max period is 2.15 mins

Question 5

Questions on probability density functions are usually well attempted. On the whole this was the case with this question, though part (ii) caused problems for some candidates.

Part (i) was well attempted; candidates must realise that when an answer is given it is important that all necessary steps in working are clearly shown. There were a few occasions here where marks were withheld for lack of essential working, but the majority of candidates gave a convincing demonstration that k was $3/8$.

In part (ii) many candidates misinterpreted the situation, and as a result their integral limits were incorrect for the probability 0.2. The information given meant that the integral of $f(x)$ from d to 2 was equal to 0.2, or the integral from 0 to d was equal to 0.8. Of the candidates who used the correct integral and formed a correct equation, the integration led to a cubic expression which many candidates failed to solve. The more successful candidates performed the integration by leaving the expression in the form $(x - 2)^2$, rather than expanding brackets, so that the resulting cubic was in the form $(d - 2)^3$.

In part (iii) candidates were required to integrate $xf(x)$. This was well attempted. The majority of candidates expanded brackets in order to perform the integration, whilst a few attempted to integrate by parts.

Answer: 0.830

$1/2$

Question 6

This question was not well attempted, as is often the case with questions involving Type I and Type II errors. There were frequent incorrect interpretations of Type I and Type II errors in the given context, despite the fact that candidates often quoted correct text book definitions. It was necessary to realise here that a Type I error would occur if Luigi's belief was rejected with $p = 0.7$, and that a Type II would occur if Maria's belief was rejected with $p = 0.35$. Thus, in part (i) the Type I error was found by calculating $1 - P(\geq 4 \text{ assuming } p = 0.7)$, or equivalent. Many candidates incorrectly found $P(\geq 4 \text{ assuming } p = 0.7)$. In part (ii), the Type II error was found by calculating $P(\geq 4 \text{ assuming } p = 0.35)$, or equivalent; many candidates found $1 - (\geq 4 \text{ assuming } p = 0.35)$. Some candidates did not use a Binomial distribution, but attempted Normal or Poisson.

In part (iii) some candidates successfully stated that a Type I error might be made, but very few were able to successfully explain why.

Answer: 0.256

0.117

Type I; they will reject Luigi's belief, although it may be true.

Question 7

This question was well attempted, with many candidates able to score highly. It was pleasing to note that the calculation of the variance in both parts was attempted more successfully than has been the case on similar questions in the past. In part (i) the probability of less than 11 using $N(10.61, 0.1017)$ was required, and in part (ii) $P(K - 1.2A > 0)$ using $N(-0.324, 0.121104)$ or equivalent was required.

In part (ii) many candidates tried to find $P(1.2K - A > 0)$ or $P(K - A > 1.2)$. It is important in such questions that candidates read the information given in the question particularly carefully.

Answer: 0.889

0.176

MATHEMATICS

Paper 9709/72

Paper 72

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. In general, candidates scored well on **Question 3(i)**, **Question 4(i)**, **Question 5(i)** and **(iii)** and **Question 7**, whilst **Question 6** proved particularly demanding.

Timing did not appear to be a problem for most candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many good and complete answers.

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Many candidates realised that the appropriate approximating distribution was Po ($1/30$). However, in many cases marks were lost due to premature rounding. Candidates used 0.03 or 0.033 rather than keeping to $1/30$ or 0.0333. It is important that candidates are aware of the accuracy level required. If $1/30$ was not stated and only 0.3 seen, this would have caused the loss of 2 marks.

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Answer: Smallest n is 20

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No evidence that the mean distance changed.

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Answer: 0-469

Max period is 2.15 mins

Question 5

Questions on probability density functions are usually well attempted. On the whole this was the case with this question, though part (ii) caused problems for some candidates.

Part (i) was well attempted; candidates must realise that when an answer is given it is important that all necessary steps in working are clearly shown. There were a few occasions here where marks were withheld for lack of essential working, but the majority of candidates gave a convincing demonstration that k was $3/8$.

In part (ii) many candidates misinterpreted the situation, and as a result their integral limits were incorrect for the probability 0.2. The information given meant that the integral of $f(x)$ from d to 2 was equal to 0.2, or the integral from 0 to d was equal to 0.8. Of the candidates who used the correct integral and formed a correct equation, the integration led to a cubic expression which many candidates failed to solve. The more successful candidates performed the integration by leaving the expression in the form $(x - 2)^2$, rather than expanding brackets, so that the resulting cubic was in the form $(d - 2)^3$.

In part (iii) candidates were required to integrate $xf(x)$. This was well attempted. The majority of candidates expanded brackets in order to perform the integration, whilst a few attempted to integrate by parts.

Answer: 0.830

$1/2$

Question 6

This question was not well attempted, as is often the case with questions involving Type I and Type II errors. There were frequent incorrect interpretations of Type I and Type II errors in the given context, despite the fact that candidates often quoted correct text book definitions. It was necessary to realise here that a Type I error would occur if Luigi's belief was rejected with $p = 0.7$, and that a Type II would occur if Maria's belief was rejected with $p = 0.35$. Thus, in part (i) the Type I error was found by calculating $1 - P(\geq 4 \text{ assuming } p = 0.7)$, or equivalent. Many candidates incorrectly found $P(\geq 4 \text{ assuming } p = 0.7)$. In part (ii), the Type II error was found by calculating $P(\geq 4 \text{ assuming } p = 0.35)$, or equivalent; many candidates found $1 - P(\geq 4 \text{ assuming } p = 0.35)$. Some candidates did not use a Binomial distribution, but attempted Normal or Poisson.

In part (iii) some candidates successfully stated that a Type I error might be made, but very few were able to successfully explain why.

Answer: 0.256

0.117

Type I; they will reject Luigi's belief, although it may be true.

Question 7

This question was well attempted, with many candidates able to score highly. It was pleasing to note that the calculation of the variance in both parts was attempted more successfully than has been the case on similar questions in the past. In part (i) the probability of less than 11 using $N(10.61, 0.1017)$ was required, and in part (ii) $P(K - 1.2A > 0)$ using $N(-0.324, 0.121104)$ or equivalent was required.

In part (ii) many candidates tried to find $P(1.2K - A > 0)$ or $P(K - A > 1.2)$. It is important in such questions that candidates read the information given in the question particularly carefully.

Answer: 0.889

0.176

MATHEMATICS

Paper 9709/73

Paper 73

Key Messages

- It is important for candidates to read the questions carefully, and answer as required.
- Candidates should check working to ensure high accuracy and to avoid arithmetic errors.

General Comments

Many candidates showed a sound understanding of the syllabus and answered the questions in an efficient manner. In many cases work was well presented and easy to follow.

Many candidates scored well on **Questions 4(ii), 6(i) and 6(ii)**, whilst some candidates found **Questions 3(ii), 4(ib) and 5(ii)** more demanding. **Question 2(ii)** required knowledge of when it is necessary to use the Central Limit Theorem. **Question 5(iii)** required candidates to apply the definition of a Type II error to the context of the given situation.

Most candidates attempted all of the questions in the available time.

Comments on Specific Questions

Question 1

Many candidates found the unbiased estimates of the population mean and variance of X correctly. Some candidates found only the biased sample variance. Some candidates did not substitute correctly into their choice of the variance formula. Many candidates showed understanding of the overall form for a confidence interval and gave their answer as an interval in an acceptable form. Some candidates omitted the 80 or did not apply the square root correctly to their variance. Many candidates used the correct z -value of 1.96.

Answers: 1.8775 or 1.88 6.81 (1.31, 2.45) or 1.31 to 2.45

Question 2

- (i) Many candidates showed a good understanding of the application of a significance test. These candidates stated the hypotheses correctly in terms of the population mean speed or used μ , standardised using the normal distribution of means of samples, compared their test statistic to the critical value clearly and stated their conclusion in context. Some candidates made errors in this process by omitting the hypotheses, or omitting the 75, or by comparing to the wrong critical values. The correct z_{crit} value was -2.054 and the correct critical probability value was 0.02. Some candidates who found the correct x_{crit} value of 59.83 omitted to compare this with 59.9. Many candidates did not state the assumption that was required to carry out the test, or stated an incorrect assumption such as “independent events”.

Answers: assume that the standard deviation is unchanged
 no evidence that the mean speed has decreased

- (ii) Many candidates realised that it was necessary to use the Central Limit Theorem, but did not give a correct reason for this. Examples of incorrect reasons were “large sample” and “population was normal”. The original distribution was not stated as normal in the question.

Answers: yes population distribution was unknown

Question 3

- (i) Most candidates attempted to find the integral of $t^2 \cdot f(t)$ with limits 0 and 10. Many candidates found this correctly as $100/3$. Some candidates did not subtract $(\text{mean } 16/3)^2$. The mean was given as $16/3$, so candidates were not required to find this.

Answer: $44/9$ or 4.89

- (ii) Many candidates attempted to connect the integral of $f(t)$ with the given 10% of patients. Some candidates did this correctly and proceeded to solve the resulting equation. Errors occurred with the use of incorrect limits or with incorrect matching of the limits and the probability. It was necessary to use “n to 10” with 0.1 or “0 to n” with 0.9. A few candidates equated to $16/3$ or to 1.282 introducing a fallacious normal distribution. Some candidates tried to use a normal distribution with mean $16/3$ and variance $44/9$. This was not a valid solution.

The correct quadratic equation in n^2 needed to be solved by the formula (or by an equivalent method). The working for this should have been shown. It was necessary to select the relevant solution (68.377), square root and round to the nearest integer (8).

Answer: 8

Question 4

- (i) (a) Many candidates correctly changed to the total goals in one match parameter 2.1 and found the probability of a total of 3 goals using the Poisson distribution. Some candidates found the probability of 0 or 1 or 2 or 3 goals in a match, which was incorrect. An alternative correct method involved finding the sum of the probabilities of scores of (0,3) and (1,2) and (2,1) and (3,0), using the given parameters 1.2 and 0.9. At this stage this method involved more work, but some of the products could be re-used in part (i)(b).

Answer: 0.189

- (i) (b) Some candidates realised that the probabilities of scores of (3,0) and (2,1) were required. Most of these candidates calculated the Poisson probabilities correctly, except for those who omitted $P(0)$ in the first product. Some candidates thought that a further conditional step was needed and divided by 0.189. This was not correct. Other candidates tried various incorrect approaches, including the use of a new Poisson distribution with parameter $\lambda = 1.2 - 0.9 = 0.3$. This was also incorrect.

Answer: 0.115

- (ii) Many candidates correctly used the normal approximation $N(30,30)$ for the new Poisson distribution $P_o(30)$ and found the correct answer. Errors made by some candidates included no continuity correction or the wrong continuity correction or an incorrect new value of λ .

Answer: 0.206

Question 5

- (i) Most candidates wrote down or calculated $E(X)$ correctly. Some candidates obtained the given answer for $\text{Var}(X)$ by showing the necessary full working. Some candidates omitted some steps or tried a binomial calculation. The binomial was not a valid method.

Answers: 3.5 $35/12$

- (ii) Many candidates showed that they understood the idea of what result would generate a Type I error, namely a mean score less than 3 when the true mean was actually 3.5. Only some candidates were able to apply this to the appropriate distribution. This was the normal distribution of means $N(\mu, \sigma^2 / n)$ with the values of μ and σ^2 as in part (i) and $n = 50$. Then $P(< 3)$ required the “tail probability” to find the $P(\text{Type I error})$. Some other errors appeared even when this correct approach was followed. For example, incorrect expressions such as $\frac{35/12}{\sqrt{50}}$ or $\frac{\sqrt{35/12}}{50}$ instead of the correct $\sqrt{\frac{35/12}{50}}$. Some candidates tried to use a binomial distribution $B(50, 1/6)$ and found $P(0 \text{ or } 1 \text{ or } 2)$. Other candidates tried to use a normal distribution $N(25/3, 125/18)$. These were incorrect methods.

Answer: 0.0192

- (iii) For a Type II error two circumstances were needed. One was that the die was biased. The second was that the experiment of 50 throws gave a mean score greater than or equal to 3. Some candidates stated both of these. Some candidates stated only one. Some candidates had the circumstances reversed and therefore incorrect.

Answers: die is biased mean of 50 throws ≥ 3

Question 6

- (i) Many candidates found the mean and variance for the total lifetime of 5 Longlive bulbs correctly. In particular the variance was calculated as $5 \times 45^2 = 10125$, sometimes by referring to $L_1 + L_2 + \dots + L_5$ which was a sensible approach. The probability that the total was less than 5200 required the larger area from the normal distribution. Some candidates made the error of finding $5^2 \times 45^2$. Some candidates incorrectly gave the smaller “tail” area as their answer. A diagram can help with this.

Answer: 0.840

- (ii) Many candidates realised that it was necessary to create a new variable ($E - 3L$) or equivalent. Some candidates correctly found the mean (-260) and variance (20929) for this variable. Other candidates found incorrect values for the new variance by subtracting variances or by using 3 instead of 3^2 . The required probability was given by the smaller “upper tail” in this part.

Answer: 0.0361 or 0.0362